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MASTER THESIS

REPORT

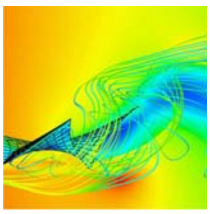
Title

The implementation of mathematical models for glass transmission in a finite element based thermal solver

Submitted by

Ceyhun BAHTIYAR

Int. Master's Program on Computational Mechanics



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Abstract

The implementation of mathematical models for glass transmission in a finite element based thermal solver

In the thesis the calculation of solar radiation coefficients, transmission-absorption-reflection, for the glazing materials are explained. Surface and layer coefficients are investigated and their physical-mathematical backgrounds are discussed. During the solution process two methods which are ray tracing and net radiation methods are introduced. The advantages and disadvantages of the both methods are presented. Finally a software tool is programmed for any possibilities of glazing systems. With theoretical examples which are solved by the software tool the subject is tried to be clarified.

Keywords: transmission-reflection-absorption; solar radiation; single and multilayer glazing; wave attenuation; wave interference; ray tracing method; net radiation method

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Introduction

The thesis is offered by P+Z Engineering. P+Z Engineering is a company within the PCL group. Engineering services for the product development are given to customers from automotive, aerospace and transportation industries by the company. The topic of the thesis is the implementation of mathematical models for glass transmission in a finite element based thermal solver.

When a car is thought either it is in motion or stationary the environmental factors warms up it in time. And this phenomena can give the damage to the some important parts of the automobile. For that reason during the design of these portions the correct heat which is transferring into the car must be calculated. One of the important factor that causes the rising of the heat is sun. So the solar radiation which goes into the car from the windows must be known exactly. The aim of the thesis is calculation of solar radiation coefficients for the glazing materials.

In the thesis there are two main goal. The first one is to find the most accurate transmission, reflection, and absorption coefficients for any kind of glazing systems. The second one is after obtained the sufficient theoretical principles creating a software tool for practical usage. As a starting point in order to obtain these aims the following factors of influence are decided to consider in a material model; thickness, angle of incidence, wavelength, single or multilayered glass, refractive index and extinction coefficient of materials, and additional influences like temperature or fluid velocity.

Firstly, the mechanism at the surface of glazing materials will be investigated. It will be determined that which of the influence factors decided before are affected the surface properties. In order to obtain surface reflection and transmission coefficient the electromagnetic wave theory will be used. By considering the electric and magnetic field of the wave either in the different glazing materials or in the air, solar radiation coefficients will be calculated at the surface of films. Importance of the wavelength will be tried to understand with the concept of the electromagnetic wave theory. Then if there is wavelength dependency solar spectrum will be our topic of discussion. Then, inside of the glazing materials will be discussed. At that point it is thought that there will be very important factors that must not be ignored. Principally these factors will be tired to be found.

Since our aim is to calculate overall system solar radiation coefficients the information obtained from the surface and inside of the layers are combined for glazing systems which are consists of different layers. In the literature there are some methods for different conditions of the systems. These methods will be investigated. Then advantages and disadvantages are going to be compared. Finally the most sufficient one will be chosen and its result will be used during the programming.

Finally, the thesis will be concluded by the programming a software tool. In the programming process C++ and Qt are going to be used. This tool is calculated system transmission and reflection for every condition that user needs. The user chooses the suitable theory and gives the required inputs to the tool and calculate transmission, reflection, and absorption coefficients.

Theory

1 Prediction of Radiative Properties by Classical Electromagnetic Theory

The relation between electric and magnetic fields and the realization that electromagnetic waves propagate with the speed of light, indicating that light itself is in the form of an electromagnetic wave. Although quantum effects have since been shown to be the controlling phenomena in electromagnetic energy propagation, it is possible to characterize many of the properties of light and radiant heat by the classical wave approach. The reflectivity, emissivity, and absorbtivity of materials can be calculated from their optical and electrical properties.

The relations between radiative, optical and electrical properties are developed by considering wave propagation in a medium and the interaction when an electromagnetic wave travelling through one medium is incidence on the surface of another medium[4]. The analysis is usually for an ideal interaction between *the incident wave and the surface*. An ideal interaction means that the results are for optically smooth, clean surfaces. The wave surface interaction is investigating by using Maxwell's equations relating electric and magnetic fields. For ideal surface conditions it is possible to perform more accurate property computations but this is usually not justified because neither the simplified nor more sophisticated approaches to an ideal interaction account for the effects of surface condition. The departures of real materials from the ideal conditions assumed in the theory are often responsible for large variations of measured property values from theoretical predictions. These departures are caused by factors such as surface roughness, surface contamination, impurities and crystal structure modification by surface working.

1.1 Electromagnetic Equations

Maxwell's equations can be used to describe the interaction of electric and magnetic fields within an isotropic medium, including vacuum, under the condition of no accumulation of static charge.

$$\nabla \times \mathrm{H} = \gamma \frac{\partial \mathrm{E}}{\partial t} + \frac{\mathrm{E}}{r_e} \qquad (1.1)$$

$$\nabla \times \mathrm{E} = -\mu \frac{\partial \mathrm{H}}{\partial t} \qquad (1.2)$$

$$\nabla \bullet \mathrm{E} = 0 \qquad (1.3)$$

$$\nabla \bullet \mathrm{H} = 0 \qquad (1.4)$$

where **H** and **E** are the magnetic and electric intensities, t is time, γ is the permittivity, r_e is the electrical resistivity, and μ is the magnetic permeability of the medium. The SI units for these quantities are in Table 1[4]. Zero subscripts denote quantities in vacuum.

The solutions to these equations reveal how radiant waves travel within a material and how the electrical and magnetic fields interact. By knowing how waves move in each of two adjacent media and applying coupling relations at the interface, relations governing absorption, transmission and reflection are obtained.

Symbol	**Quantity**	**Units**	**Value**
C	Speed of electromagnetic wave propagation	m/s	c_0 =2.997925*10^8
E	Electric intensity	N/C; C/(m*s)	
H	Magnetic intensity	Ω*m; N*m^2*s/C^2	
S	Instantaneous rate of energy transport per unit area		
γ	Electrical permittivity	C^2/(N*m^2)	$\gamma_0 = \dfrac{1}{\mu_0 c_0} = 8.85419\times10^{-12}$
μ	Magnetic permeability	N*s^2/C^2	$\mu_0 = 4\pi\times10^{-7}$

*Table 1 **:** Quantities for use in electromagnetic equations in SI units(a quantity with a zero subscript is in vacuum) [4]*

Propagations are considered within an infinite, homogeneous, isotropic medium. First perfect dielectric (non-conductor, perfect insulator) is considered. The electromagnetic waves do not attenuate in such a material. Than imperfect dielectrics (poor conductors) are analyzed. They are the media that has finite electrical conductivity and waves do attenuate in these materials because they absorb energy.[4]

1.1.1 Propagation in Perfect Dielectric Media

In perfect dielectric media electric resistivity is so large.[4] For that reason $\mathbf{E}/r_e$ term can be neglected in Eq.(1.1). Equations (1.1) and (1.2) can then be written in Cartesian coordinates to provide two sets of three equations relating the x, y, and z components of the electric and magnetic intensities,

$$\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = \gamma \frac{\partial E_x}{\partial t} \tag{1.5}$$

$$\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = \gamma \frac{\partial E_y}{\partial t} \tag{1.6}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = \gamma \frac{\partial E_z}{\partial t} \tag{1.7}$$

$$\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} = -\mu \frac{\partial H_x}{\partial t} \tag{1.8}$$

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t} \tag{1.9}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\mu \frac{\partial H_z}{\partial t} \tag{1.10}$$

From Equations (1.3) and (1.4),

$$\frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = 0 \tag{1.11}$$

$$\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y} + \frac{\partial H_z}{\partial z} = 0 \tag{1.12}$$

The coordinate system is fixed to the path of a wave propagating in the x direction. Plane wave is considered where all the quantities concerned with the wave are constant over yz plane at any time,

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0 \tag{1.13}$$

and Equations (1.5) to (1.12) reduce to,

$$0 = \gamma \frac{\partial E_x}{\partial t} \tag{1.14}$$

$$-\frac{\partial H_z}{\partial x} = \gamma \frac{\partial E_y}{\partial t} \tag{1.15}$$

$$\frac{\partial H_y}{\partial x} = \gamma \frac{\partial E_z}{\partial t} \tag{1.16}$$

$$0 = -\mu \frac{\partial H_x}{\partial t} \tag{1.17}$$

$$-\frac{\partial E_z}{\partial x} = -\mu \frac{\partial H_y}{\partial t} \tag{1.18}$$

$$\frac{\partial E_y}{\partial x} = -\mu \frac{\partial H_z}{\partial t} \tag{1.19}$$

$$\frac{\partial E_x}{\partial x} = 0 \tag{1.20}$$

$$\frac{\partial H_x}{\partial x} = 0 \tag{1.21}$$

The H components are then eliminated. After differentiating Eq.(1.15) with respect to t, and (1.19) with respect to x, the results are combined to eliminate H_z. Similarly, from (1.16) and (1.18) the H_y is eliminated. This yields,

$$\mu\gamma \frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2} \tag{1.22}$$

$$\mu\gamma \frac{\partial^2 E_z}{\partial t^2} = \frac{\partial^2 E_z}{\partial x^2} \tag{1.23}$$

These wave equations govern the propagation of E_y and E_z in the x direction. For simplicity in derivation, it is assumed that the electromagnetic waves are polarized such that the vector **E** is in the xy plane (Figure 1). Then E_z and its derivatives are zero. But later both E_y and E_z will be considered during calculation of reflectance, transmission, and absorption

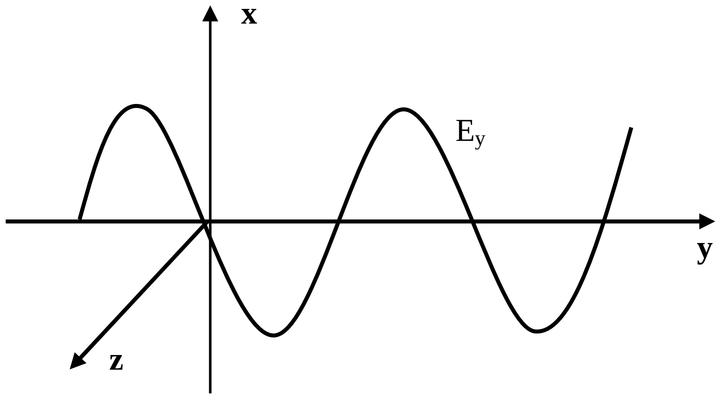


*Figure 1 **:** Electric field polarized in the xy plane[2]*

The vector **E** has only x and y components. For the x components of **E** and **H**,

$$\frac{\partial E_x}{\partial t} = \frac{\partial E_x}{\partial x} = \frac{\partial H_x}{\partial t} = \frac{\partial H_x}{\partial x} = 0 \tag{1.24}$$

The electric and magnetic intensity components in the direction of propagation are steady and independent of the propagation direction x. The only time-varying component of **E** is E_y. Since this component is normal to the x direction of propagation, the wave is transverse wave.[4]

Equation (1.22) is the wave equation for propagation of E_y in the x direction. Its general solution:

$$E_y = f(x - \frac{t}{\sqrt{\mu\gamma}}) + g(x + \frac{t}{\sqrt{\mu\gamma}}) \tag{1.25}$$

where f and g are any differentiable functions. Here $1/\sqrt{\mu\gamma}$ is speed of propagation in the medium. The f provides propagation in the positive x direction, while g accounts for propagation in the negative x direction. So only the f function is considered.

$$E_y = f(x - \frac{t}{\sqrt{\mu\gamma}}) \tag{1.26}$$

represents a wave with y component E_y, propagating in the positive x direction with speed $1/\sqrt{\mu\gamma}$. In vacuum, the propagation speed is c_0, so there is the relation $c_0 = 1/\sqrt{\mu_0\gamma_0}$.

Accompanying the E_y wave is a companion magnetic wave. If Eq.(1.15) is differentiated with respect to x and (1.19) with respect to t, the result can be combined to yield

$$\mu\gamma \frac{\partial^2 H_z}{\partial t^2} = \frac{\partial^2 H_z}{\partial x^2} \qquad (1.27)$$

which is the same wave equation as (1.22). As a result, the H_z component of the magnetic field propagates along with E_y as in Figure 2.

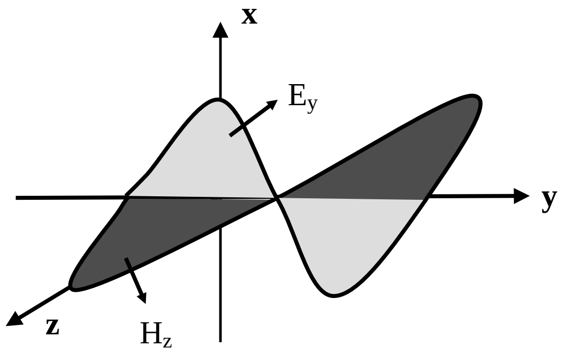


*Figure 2 **:** Electric field polarized in the xy plane, travelling in the x direction with companion magnetic field wave[4]*

Any propagating wave form as designated by the f function can be represent using Fourier series as a superposition of waves, each having a different fixed wavelength. Consider only one such spectral wave and note that any waveform could be build up from a number of spectral components. And at the origin x=0 let the waveform variation with time be

$$E_y = E_{yM} \exp(i\omega t) = E_{yM} (\cos \omega t + i \sin \omega t) \qquad (1.28)$$

A position on the wave that leaves the origin at time t_1 arrives at the location x after a time x/c, where c is the wave speed in the medium. Hence the time of arrival is,

$$t = t_1 + x / c \qquad (1.29)$$

so that the time of leaving the origin was,

$$t_1 = t - x / c \qquad (1.30)$$

A wave travelling in the positive x direction is then given by

$$E_y = E_{yM} \exp[i\omega(t - \frac{x}{c})] = E_{yM} \exp[i\omega(t - \sqrt{\mu\lambda}x)] \qquad (1.31)$$

If desired, $\omega = 2\pi\nu = 2\pi c / \lambda_m = 2\pi c_0 / \lambda_0$ can be written in the above equation. As explained before the simple refractive index n is defined as the ratio of the wave speed in vacuum c_0 to the speed in the medium $c = 1/\sqrt{\mu\gamma}$. Hence, $n = c_0 / c = c_0\sqrt{\mu\gamma}$, and (1.31) can be written as,

$$E_y = E_{yM} \exp[i\omega(t - \frac{n}{c_0} x)] \qquad (1.32)$$

The wave propagates with undiminished amplitude in the ideal medium. This is a consequence of the medium being a perfect dielectric, that is, having zero electrical conductivity. In many real materials the electrical conductivity is significant and the last term on the right in (1.1) cannot be neglected; this causes the propagating wave to attenuate.[4]

1.1.2 Wave Propagation with Finite Electrical Conductivity

For simplicity a single plane wave is again considered. In these materials (imperfect dielectrics) there is attenuation of a travelling wave. If an exponential attenuation with distance is introduced, the wave takes the form

$$E_y = E_{yM}\exp[i\omega(t - \frac{n}{c_0}x)]\exp(-\frac{\omega}{c_0}\kappa x)] \tag{1.33}$$

where κ is the extinction coefficient of the wave in the medium. The attenuation term indicates there is energy absorption from the wave as it travels through the medium. If the exponential terms are combined into the relation,

$$E_y = E_{yM}\exp\{i\omega[t - (n - i\kappa)\frac{x}{c_0}]\} \tag{1.34}$$

A comparison of (1.32) with (1.34) shows that the simple refractive index n is replaced in an attenuating medium by the complex refractive index $\overline{n}$,

$$\overline{n} = n - i\kappa \tag{1.35}$$

Equation (1.34) is a solution of the governing equations with the last term on the right of (1.1) included. With this term retained, (1.22) takes the form,

$$\mu\gamma\frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2} - \frac{\mu}{r_e}\frac{\partial E_y}{\partial t} \tag{1.36}$$

The wave amplitude in (1.34) is substituted into (1.36) and the equality results,

$$c_0^2\mu\gamma = (n - i\kappa)^2 + \frac{i\mu\lambda_0 c_0}{2\pi r_e} \tag{1.37}$$

which is a relation necessary for the wave to satisfy Maxwell's equations. Equating the real and imaginary parts of (1.37) yields,

$$n^2 - \kappa^2 = \mu\gamma c_0^2 \tag{1.38}$$

$$n\kappa = \frac{\mu\lambda_0 c_0}{4\pi r_e} \tag{1.39}$$

These equations are solved for the components of the complex refractive index

$$n^2 = \frac{\mu\gamma c_0^2}{2}\{1 + [1 + (\frac{\lambda_0}{2\pi c_0 r_e\gamma})^2]^{1/2}\} \tag{1.40}$$

$$\kappa^2 = \frac{\mu\gamma c_0^2}{2}\{-1 + [1 + (\frac{\lambda_0}{2\pi c_0 r_e\gamma})^2]^{1/2}\} \tag{1.41}$$

It can be easily seen that both the refractive index(n(λ_0)) and extinction coefficient(κ(λ_0)) depends on the wavelength.[4]

Comparison of (1.32) for perfect insulating media with (1.34) for the conducting media shows the solutions to be identical in form with the simple refractive n for the perfect insulator solution replaced, for conductors, by the complex refractive index $\overline{n} = n - i\kappa$. This means that some of the relations for perfect dielectrics will apply for conductors, provided $n - i\kappa$ is substituted for n.

1.1.3 Energy of an Electromagnetic Wave

The energy carried per unit time and per unit area by an electromagnetic wave is given by the cross product of the electric and magnetic intensity factors.

$$\mathbb{S} = \mathrm{E} \times \mathrm{H} \tag{1.42}$$

S is a vector propagating at right angles to the **E** and **H** vectors in a direction defined by the right hand rule. For the plane wave in Figure 2 the propagation is in the positive x direction, and the magnitude of **S** is,

$$|\mathbb{S}| = E_y H_z \tag{1.43}$$

If E_y is given by (1.34) and (1.19), which applies for conductors, can be used to find H_z as follows,

$$-\mu \frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} = \frac{-i\omega}{c_0}(n - i\kappa)E_y = -\frac{i\omega\overline{n}}{c_0}E_y \tag{1.44}$$

Noting the t dependence of E_y in (1.34) and integrating above equation yield,

$$H_z = \frac{\overline{n}}{\mu c_0} E_y \tag{1.45}$$

The integration constant has been set equal to zero. The constant corresponds to the presence of a steady magnetic intensity in addition to that induced by E_y and is assumed to be zero.

When H_z is substituted in (1.43), the magnitude of the **S** (Poynting vector) becomes,

$$|\mathbb{S}| = \frac{\overline{n}}{\mu c_0} E_y^2 \tag{1.46}$$

which has more general vector form,

$$|\mathbb{S}| = \frac{\overline{n}}{\mu c_0} |\mathrm{E}|^2 \tag{1.47}$$

Thus, the energy per unit time and area carried by the wave is proportional to the square of the amplitude of the electric intensity. For radiation passing through a medium, the exponential decay factor in the spectral intensity must, by virtue of (1.46), be equal to the square of the decay term in E_y. Thus, from (1.33), the intensity decay factor is,

$$\tau = \exp(-2\omega\kappa \frac{x}{c_0}) \quad \text{or} \quad \tau = \exp(-4\pi\kappa \frac{x}{\lambda_0}) \tag{1.48}$$

As a result of absorption the intensity decays as,

$$\tau = \exp(ax) \tag{1.49}$$

The above three equations (1.48) and (1.49) determine the how much amount of energy will be transmitted along the travelling distance of light(x), under the effect of absorption. Here \`a´ is the absorption coefficient. So in order to calculate absorbing energy the wave has to travel some thickness in the material. And on the material surfaces(zero thickness) there becomes no absorption.

Thus there is the very useful relation between a and κ given by,

$$a(\lambda_0) = \frac{4\pi\kappa(\lambda_0)}{\lambda_0} \tag{1.50}$$

This provides a means for obtaining the spectral absorption coefficient $a(\lambda_0)$ from optical data for the extinction coefficient $\kappa(\lambda_0)$ as a function of wavelength.[4]

1.2 Laws of Reflection and Refraction

Up to now the wave nature of propagating radiation and the characteristics of movement through infinite homogeneous isotropic media have been found. Now the interaction of an electromagnetic wave with the interface between two media is considered. This provides laws for reflection and refraction in terms of the indices of refraction and extinction coefficients.

1.2.1 Between Two Perfect Dielectric(No wave Attenuation, κ≈0) (No wave interference effect)

The interaction is considered at an optically smooth interface between two non-attenuating (perfect dielectric) materials. For simplicity a simple cosine wave is used.[4] So real part of the below equation is considered,

$$E_y = E_{yM} \exp[i\omega(t - \frac{n}{c_0}x)] \tag{1.51}$$

An x', y', z' coordinate system is fixed to the path of the incident wave that is moving in the x' direction. The wave strikes the interface as in Figure 3. The interface is in the yz plane of the x, y, z coordinates attached to the media. The *plane of incidence*(Figure 3) is defined by containing both the normal to the interface and the incidence direction on x'. The coordinate system has been drawn so the y' direction is in the plane of incidence. The interaction of the wave with the interface depends on the wave orientation relative to the plane of incidence.[4]

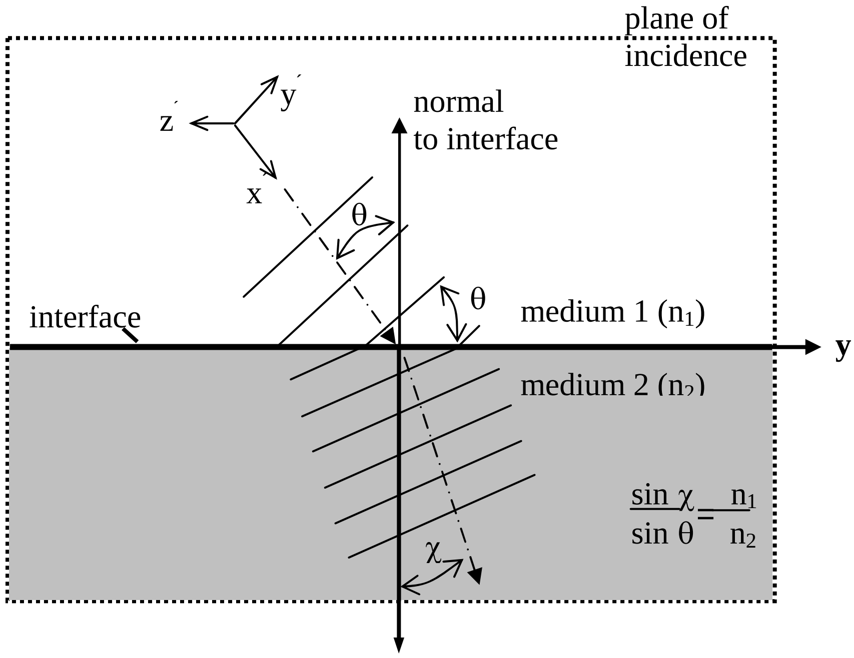


Figure 3 : Plane wave incident upon interface between two media

Consider an incident wave $E_{\|,i}$ polarized so that it has amplitude only in the x' y' plane and hence is parallel to the plane of incidence(Figure 4). Because incident wave electric field polarization, $E_{\|,i}$, is in y' direction, the corresponding magnetic field will be in the z' direction (Figure 4).

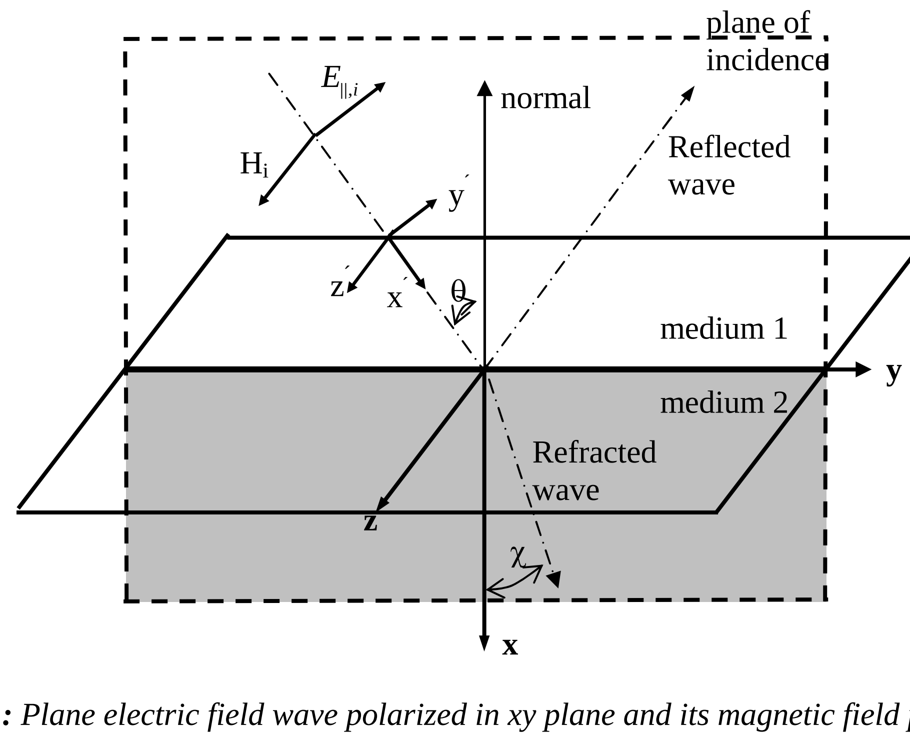


Figure 4 : Plane electric field wave polarized in xy plane and its magnetic field polarization

From Equation (1.32) retaining only the real part (cosine term), the wave propagating in the x' direction is defined by:

$$E_{\|,i} = E_{M\|,i} \cos[\omega(t - \frac{n_1}{c_0} x')] \tag{1.52}$$

From Figure 5, the components of the plane electric field incident wave, which is polarized in the parallel of the plane of incidence, in the x,y,z coordinate system are :

$$E_{x,i} = E_{\|,i}\sin(\theta) \qquad E_{y,i} = E_{\|,i}\cos(\theta) \qquad E_z = 0 \tag{1.53}$$

Substituting (1.52) into (1.53) and noting that x', the distance the wave travels in a given time, is related to the y distance travels along the interface by $x' = y\sin\theta$, we obtain for the incident components,

$$E_{x,i} = -E_{M\|,i}\sin\theta\cos[\omega(t - \frac{n_1 y\sin\theta}{c_0})] \tag{1.54}$$

$$E_{y,i} = E_{M\|,i}\cos\theta\cos[\omega(t - \frac{n_1 y\sin\theta}{c_0})] \tag{1.55}$$

$$E_{z,i} = 0 \tag{1.56}$$

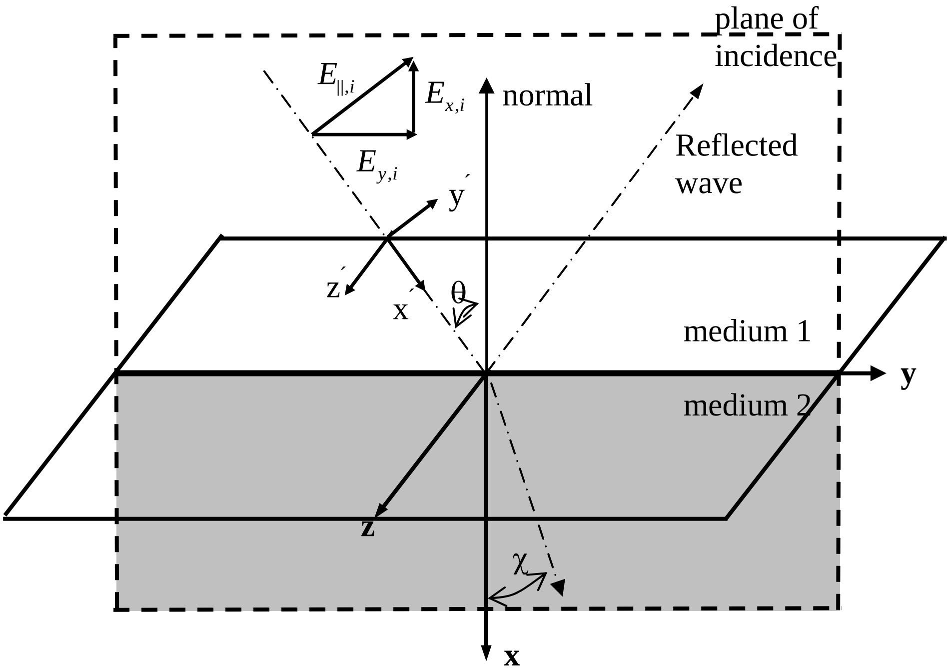


Figure 5 : The components of the plane electric field (polarized in parallel to plane of incidence) incident wave in the x, y, z coordinate system[4]

When the wave strikes the interface, the wave separates into $E_{\|,r}$ reflected at angle θ_r, and $E_{\|,t}$ transmitted at angle χ(see Figure 6).

The reflected components,

$$E_{x,r} = -E_{M\|,r}\sin\theta_r\cos[\omega(t - \frac{n_1 y\sin\theta_r}{c_0})] \tag{1.57}$$

$$E_{y,r} = -E_{M\|,r}\cos\theta_r\cos[\omega(t \quad \frac{n_1 y\sin\theta_r}{c_0})] \tag{1.58}$$

$$E_{z,r} = 0 \tag{1.59}$$

The transmitted components,

$$E_{x,t} = -E_{M\|,t} \sin\chi \cos[\omega(t - \frac{n_2 y \sin\chi}{c_0})] \quad (1.60)$$

$$E_{y,t} = E_{M\|,t} \cos\chi \cos[\omega(t - \frac{n_2 y \sin\chi}{c_0})] \quad (1.61)$$

$$E_{z,t} = 0 \quad (1.62)$$

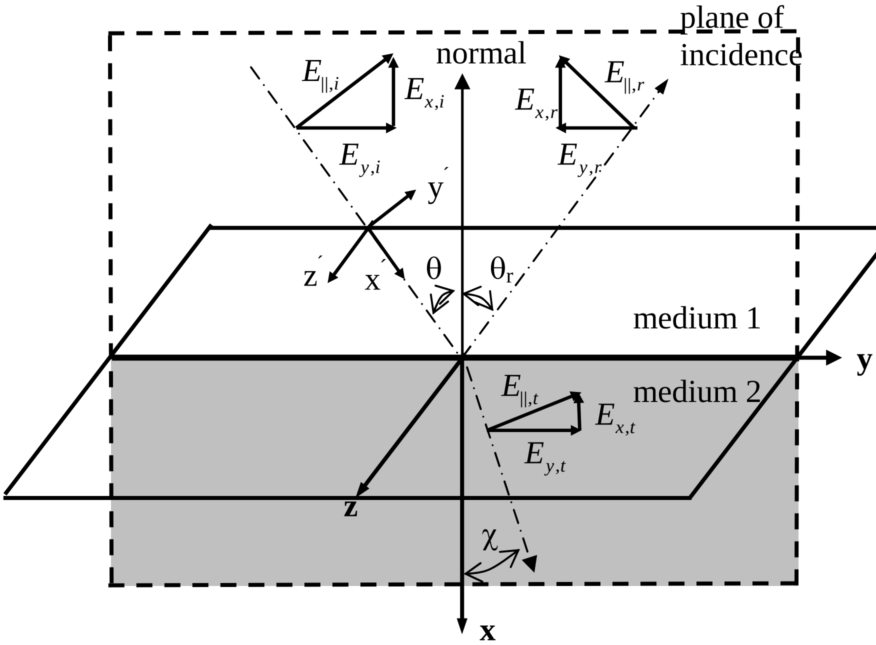


Figure 6 : The components of the plane electric field (polarized in parallel to plane of incidence) incident, refracted, and reflected waves in the x,y,z coordinate system[4]

Certain boundary conditions must be satisfied at the interface of the two media. The sum of the components, parallel to the interface, of the electric intensities of the reflected and incident waves must equal the intensity of the refracted wave in the same plane. Because the intensity in medium 1 is the superposition of the incident and reflected intensities. For the polarized wave, this condition gives the following equality for the y components that are parallel to the interface,

$$\{E_{M\|,i} \cos\theta \cos[\omega(t - \frac{n_1 y \sin\theta}{c_0})] - E_{M\|,r} \cos\theta_r \cos[\omega(t - \frac{n_1 y \sin\theta_r}{c_0})] = E_{M\|,t} \cos\chi \cos[\omega(t - \frac{n_2 y \sin\chi}{c_0})]\}_{x=0} \quad (1.63)$$

The above equation must hold for arbitrary t and y and the angles θ, θ_r and χ are independent of t and y, so the cosine terms involving time must be equal. This is true only if,

$$n_1 \sin\theta = n_1 \sin\theta_r = n_2 \sin\chi \quad (1.64)$$

which provides,

$$\theta = \theta_r \qquad \text{and} \qquad \frac{\sin\chi}{\sin\theta} = \frac{n_1}{n_2} \tag{1.65}$$

Equation (1.65) relates angle of refraction and the angle of incidence by means of the ratio of refractive indices and is known as *Snell's law*.
Because the cosine terms which are involving time are equal, Eq.(1.63) turns into form of,

$$(E_{M\|,i}\cos\theta - E_{M\|,r}\cos\theta = E_{M\|,t}\cos\chi)_{x=0} \tag{1.66}$$

The magnetic intensity parallel to the boundary(z' direction) must be continuous at the boundary plane. The magnetic intensity vector is perpendicular to the electric intensity; since the electric intensity being considered is in the plane of incidence, the magnetic intensity is parallel to the boundary(As mentioned before, because incident wave electric field polarization, $E_{\|,i}$, is in y' direction, the corresponding magnetic field will be in the z' direction(Figure 4). Because it is parallel, continuity at the boundary provides that,

$$(\mathrm{H}_i + \mathrm{H}_r = \mathrm{H}_t)_{x=0} \tag{1.67}$$

A relation between specific components H_z and E_y was derived(Eq.(1.45)). It is true more generally, so the magnitudes of the **E** and **H** vectors are related for non-attenuating materials by:

$$|\mathrm{H}| = \frac{n}{\mu c_0}|\mathrm{E}| \tag{1.68}$$

For both dielectrics and metals the magnetic permeability is very close to that of a vacuum, so that $\mu \approx \mu_0$. Then Eq.(1.67) can be written as,

$$(n_1 E_{M\|,i} + n_1 E_{M\|,r} = n_2 E_{M\|,t})_{x=0} \tag{1.69}$$

Equations (1.66) and (1.69) are combined to eliminate $E_{M\|,t}$ and give the reflected electric intensity in terms of the incident intensity for non-attenuating materials as,

$$\frac{E_{M\|,r}}{E_{M\|,i}} = \frac{\cos\theta/\cos\chi - n_1/n_2}{\cos\theta/\cos\chi + n_1/n_2} \tag{1.70}$$

Up to now electric wave which is polarized parallel to the incident plane and its corresponding magnetic field polarization which is perpendicular to the xy plane are considered. And now perpendicular electric field and parallel magnetic field wave polarizations become the discussion.

$E_{\perp,i}$ is a polarized incident wave which is in the z' direction. It has amplitude only in the $z'\,y'$ plane and hence is perpendicular to the plane of incidence and parallel to the boundary(Figure 7). So continuity at the boundary provides that,

$$E_{M\perp,i} + E_{M\perp,r} = E_{M\perp,t} \tag{1.71}$$

As mentioned before, the magnetic intensity vector is perpendicular to the electric intensity; since the electric intensity being considered is parallel to the boundary, that is perpendicular to the plane of incidence, the magnetic intensity is in the plane of incidence(Figure 7). The magnitudes of the **E** and **H** vectors are related for non-attenuating materials by,

$$|\mathrm{H}| = \frac{n}{\mu c_0}|\mathrm{E}| \tag{1.72}$$

and for both dielectrics and metals the magnetic permeability is very close to that of a vacuum($\mu \approx \mu_0$). Then equation below can be written,

$$E_{M\perp,i} n_1 \cos\theta - E_{M\perp,r} n_1 \cos\theta = E_{M\perp,t} n_2 \cos\chi \tag{1.73}$$

Equations (1.71) and (1.73) are combined to eliminate $E_{M\perp,t}$ and give the reflected electric intensity in terms of the incident intensity for non-attenuating materials as,

$$\frac{E_{M\perp,r}}{E_{M\perp,i}} = -\frac{\cos\chi/\cos\theta - n_1/n_2}{\cos\chi/\cos\theta + n_1/n_2} \tag{1.74}$$

If Snell's Law is used in Eq.(1.70) and (1.74) to eliminate n1/n2 in terms of sinχ/sinθ to yield,

$$\frac{E_{M\|,r}}{E_{M\|,i}} = \frac{\tan(\theta-\chi)}{\tan(\theta+\chi)} \qquad \frac{E_{M\perp,r}}{E_{M\perp,i}} = -\frac{\sin(\theta-\chi)}{\sin(\theta+\chi)} \tag{1.75}$$

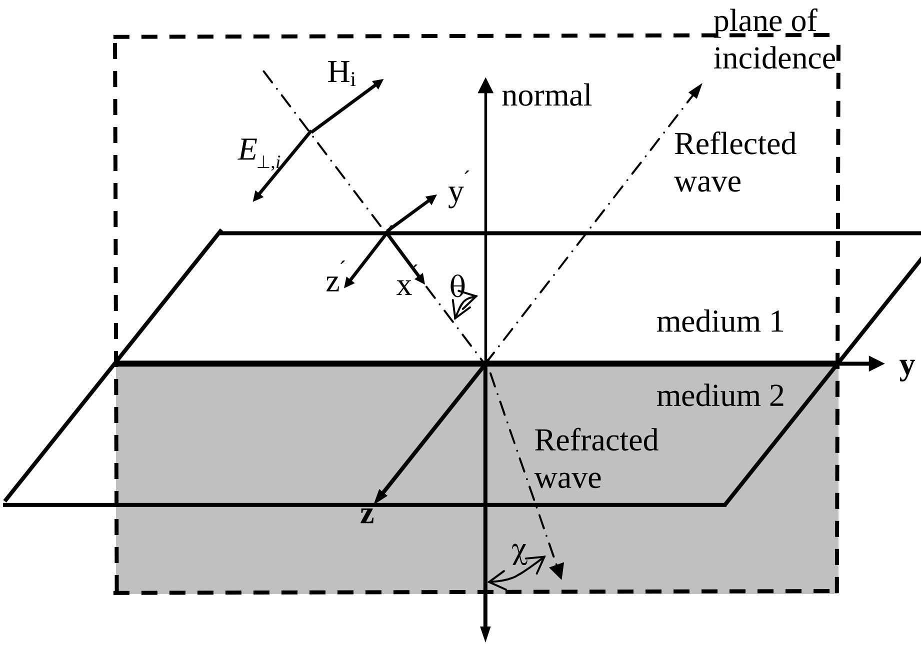


Figure 7 : Plane electric field wave polarized in perpendicular to plane of incidence and its magnetic field polarization

The energy carried by a wave is proportional to the square of the wave amplitude. So squaring $E_{M,r}/E_{M,i}$ gives the ratio of energy reflected from a surface to energy incident from a given direction. For the representation of the surface reflection is ρ. The values of $\rho(\lambda,\theta)$ for incident parallel and perpendicular polarized components are then obtained as,

$$\rho_{\|}(\lambda,\theta) = \left(\frac{E_{M\|,r}}{E_{M\|,i}}\right)^2 = \frac{\tan^2(\theta-\chi)}{\tan^2(\theta+\chi)} = \left(\frac{\cos\theta/\cos\chi - n_1/n_2}{\cos\theta/\cos\chi + n_1/n_2}\right)^2 \tag{1.76}$$

$$\rho_{\perp}(\lambda,\theta) = \left(\frac{E_{M\perp,r}}{E_{M\perp,i}}\right)^2 = \frac{\sin^2(\theta-\chi)}{\sin^2(\theta+\chi)} = \left(-\frac{\cos\chi/\cos\theta - n_1/n_2}{\cos\chi/\cos\theta + n_1/n_2}\right)^2 \tag{1.77}$$

Sum of reflection, transmission and absorption is equal to the 1. From Eq.(1.48), it can be seen that there is no absorption on the surface. Because the thickness is equal to zero. So the transmission of the surface for two kind of polarization are $1-\rho_{\perp}(\lambda,\theta)$ and $1-\rho_{\|}(\lambda,\theta)$.

The θ and χ can be interchanged and hence along the incident and refracted paths the reflection and transmission are the same for radiation incident on an interface either from the outside or from within the material.

The average of reflection of the surface,

$$\rho(\lambda,\theta)=\frac{\rho_{\|}(\lambda,\theta)+\rho_{\perp}(\lambda,\theta)}{2} \tag{1.78}$$

Equation (1.78) is Fresnel's equation, and it gives the reflectivity for an unpolarized ray incident upon an interface between two dielectric media.

When the incident radiation is normal to the interface, $\cos\theta=\cos\chi=1$, and Eq.(1.70) and (1.74) yield

$$\frac{E_{M\|,r}}{E_{M\|,i}}=-\frac{E_{M\perp,r}}{E_{M\perp,i}}=\frac{1-n_1/n_2}{1+n_1/n_2}=\frac{n_2-n_1}{n_2+n_1} \tag{1.79}$$

The normal reflectivity is then,

$$\rho_n(\lambda)=\rho(\lambda,\theta=\theta_r=0)=(\frac{n_2-n_1}{n_2+n_1})^2=[\frac{(n_2/n_1)-1}{(n_2/n_1)+1}]^2 \tag{1.80}$$

The reflectivity is spectral(depends on wavelength) because n_1 and n_2 are functions of λ.

1.2.2 Between Two Absorbing Medium (κ≠0) (No wave interference effect)

The propagation of a wave in an infinite medium that attenuates the wave is governed by the same relations as in a non-attenuating medium if the refractive index `n´ for the latter case is replaced by $\overline{n}=n-i\kappa$. When the interaction of a wave with a boundary is considered, the theoretical expressions for reflected wave amplitudes derived for non-attenuating media ($\kappa=0$) also apply for attenuating media if $\overline{n}$ is used instead of n, but this lead to some complexities.[4]

First of all Snell's law becomes,

$$\frac{\sin\chi}{\sin\theta}=\frac{\overline{n}_1}{\overline{n}_2}=\frac{n_1-i\kappa_1}{n_2-i\kappa_2} \tag{1.81}$$

Because this relation is complex, $\sin\chi$ can no longer be interpreted physically as a simple angle of refraction. Except for the special case of normal incidence, n is no longer directly related to the propagation velocity.

For the oblique incidence(Figure 8) the reflectivity can be derived by starting from (1.70) and (1.74) and using the complex index of refraction.[4] For incident rays polarized parallel or perpendicular to the plane of incidence, the below equations are obtained,

$$\frac{E_{M\|,r}}{E_{M\|,i}} = \frac{\cos\theta / \cos\chi - (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}{\cos\theta / \cos\chi + (n_1 - i\kappa_1)/(n_2 - i\kappa_2)} \tag{1.82}$$

$$\frac{E_{M\perp,r}}{E_{M\perp,i}} = -\frac{\cos\chi / \cos\theta - (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}{\cos\chi / \cos\theta + (n_1 - i\kappa_1)/(n_2 - i\kappa_2)} \tag{1.83}$$

But because of complexity of index of refraction, the above ratios are also complex. The energy carried by a wave is proportional to the square of the wave amplitude. So squaring $E_{M,r} / E_{M,i}$ gives the ratio of energy reflected from a surface to energy incident from a given direction. The values of $\rho_\lambda(\lambda,\theta)$ for incident parallel and perpendicular polarized components are then obtained as,

$$\rho_\|(\lambda,\theta) = (\frac{E_{M\|,r}}{E_{M\|,i}})^2 \tag{1.84}$$

$$\rho_\perp(\lambda,\theta) = (\frac{E_{M\perp,r}}{E_{M\perp,i}})^2 \tag{1.85}$$

For a complex quantity z, $|z|^2 = zz^*$, where z^* is the complex conjugate. So during the calculation of (1.84) and (1.85), (1.82) and (1.83) will be multiplied by their complex conjugates.

Sum of reflection, transmission and absorption is equal to the 1. From Eq.(1.48), it can be seen that there is no absorption on the surface. Because the thickness is equal to zero. So the transmission of the surface for two kind of polarization are $1-\rho_\perp(\lambda,\theta)$ and $1-\rho_\|(\lambda,\theta)$.

The θ and χ can be interchanged and hence along the incident and refracted paths the reflection and transmission are the same for radiation incident on an interface either from the outside or from within the material.

The reflectivity is spectral(depends on wavelength) because n_1, n_2, and κ are functions of λ.

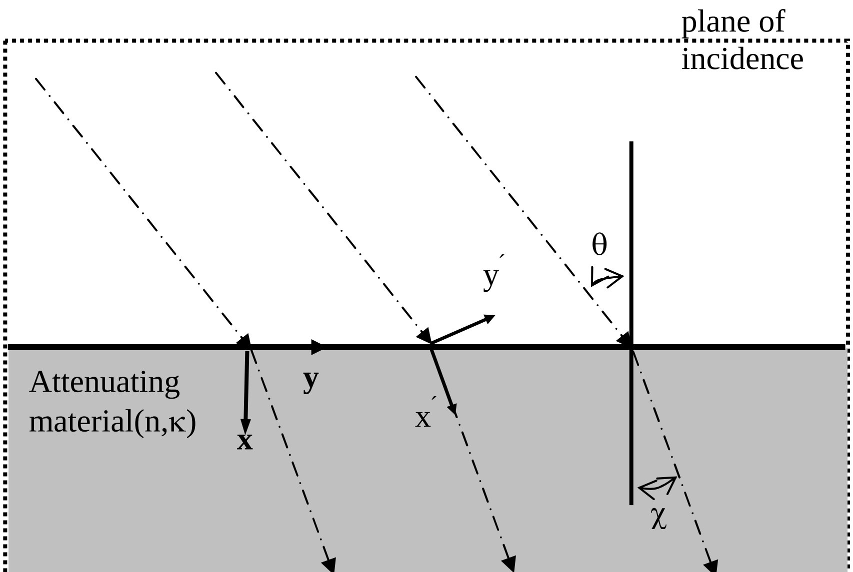


Figure 8 : Oblique incidence onto an attenuating material and propagation[4]

To clarify the discussion, below two specific condition will be considered as an example. First one is normal incidence, the second one is radiation incident in air or vacuum on a material with properties n and κ.

1.example: Because of normal incidence θ and χ are 0(zero) degree. So Equations (1.82) and (1.83) turn into form of,

$$\frac{E_{M\|,r}}{E_{M\|,i}} = -\frac{E_{M\perp,r}}{E_{M\perp,i}} = \frac{\bar{n}_2 - \bar{n}_1}{\bar{n}_2 + \bar{n}_1} = \frac{n_2 - i\kappa_2 - (n_1 - i\kappa_1)}{n_2 - i\kappa_2 + (n_1 - i\kappa_1)} \tag{1.86}$$

Then, because the relations for $\|$ and $\perp$ polarization are the same for normal incidence, the reflectivity is,

$$\rho_{\lambda,n}(\lambda) = [\frac{n_2 - i\kappa_2 - (n_1 - i\kappa_1)}{n_2 - i\kappa_2 + (n_1 - i\kappa_1)}][\frac{n_2 - i\kappa_2 - (n_1 - i\kappa_1)}{n_2 - i\kappa_2 + (n_1 - i\kappa_1)}]$$
$$= \frac{(n_2 - n_1)^2 + (\kappa_2 - \kappa_1)^2}{(n_2 + n_1)^2 + (\kappa_2 + \kappa_1)^2} \tag{1.87}$$

2.example: This is one of the important cases. For an incident ray in air, $n \approx 1$ and $\kappa \approx 0$. As mentioned before the other medium has the properties n and κ and the incident angle is arbitrary. Then, from (1.81), (1.82) and (1.83) with $\bar{n} = n - i\kappa$,

$$\frac{E_{M\|,r}}{E_{M\|,i}} = \frac{\bar{n}\cos\theta - \cos\chi}{\bar{n}\cos\theta + \cos\chi} \qquad \frac{E_{M\perp,r}}{E_{M\perp,i}} = -\frac{\bar{n}\cos\chi - \cos\theta}{\bar{n}\cos\chi + \cos\theta} \tag{1.88}$$

Snell's law becomes,

$$\frac{\sin\chi}{\sin\theta} = \frac{1}{n - i\kappa} = \frac{1}{\bar{n}} \tag{1.89}$$

The $\bar{n}\cos\chi$ in (1.89) can be found as,

$$\bar{n}\cos\chi = \bar{n}(1 - \sin^2\chi)^{1/2} = (\bar{n}^2 - \sin^2\theta)^{1/2} \tag{1.90}$$

The results are presented more conveniently by letting $a - ib = (\bar{n}^2 - \sin^2\theta)^{1/2}$. By squaring and equating real and imaginary parts, the resulting simultaneous equations are solved for a and b to obtain,

$$a^2 = \frac{1}{2}\{[(n^2 - \kappa^2 - \sin^2\theta)^2 + 4n^2\kappa^2]^{1/2} + n^2 - \kappa^2 - \sin^2\theta\} \tag{1.91}$$

$$b^2 = \frac{1}{2}\{[(n^2 - \kappa^2 - \sin^2\theta)^2 + 4n^2\kappa^2]^{1/2} - n^2 - \kappa^2 - \sin^2\theta\} \tag{1.92}$$

The quantity $a - ib$ is substituted for $\bar{n}\cos\chi$ in (1.88), and the resulting equations are multiplied through by their complex conjugates to yield the reflectivity,

$$\rho_{\perp}(\lambda,\theta) = \frac{a^2 + b^2 - 2a\cos\theta + \cos^2\theta}{a^2 + b^2 + 2a\cos\theta + \cos^2\theta} \tag{1.93}$$

$$\rho_{\|}(\lambda,\theta) = \frac{a^2 + b^2 - 2a\sin\theta\tan\theta + \cos^2\theta}{a^2 + b^2 + 2a\sin\theta\tan\theta + \cos^2\theta}\rho_{\perp}(\lambda,\theta) \tag{1.94}$$

And the reflectivity and transmission of the surface are average of the parallel and perpendicular components.[1]

1.2.3 Vector Amplitudes at the Interface between Two Perfect Dielectric(No wave Attenuation, κ≈0) (With wave interference effect)

Before starting the propagation of a wave in an infinite non-attenuating medium, wave interference effect will be considered.

Light wave interference results when two waves are travelling through a medium and meet up at the same location. When a wave (light waves included) reaches the boundary between two medium, a portion of the wave reflects off the boundary and a portion is transmitted across the boundary. The reflected portion of the wave remains in the original medium. The transmitted portion of the wave enters the new medium and continues travelling through it until it reaches a subsequent boundary (Figure 9). If the new medium is a thin film, then the transmitted wave does not travel far before it reaches a new boundary and undergoes the usual reflection and transmission behaviour. Thus, there are two waves which emerge from the film - one wave which is reflected off the top of the film (*wave a* in the Figure 9) and the other wave which reflects off the bottom of the film (*wave b* in the Figure 9).[2]

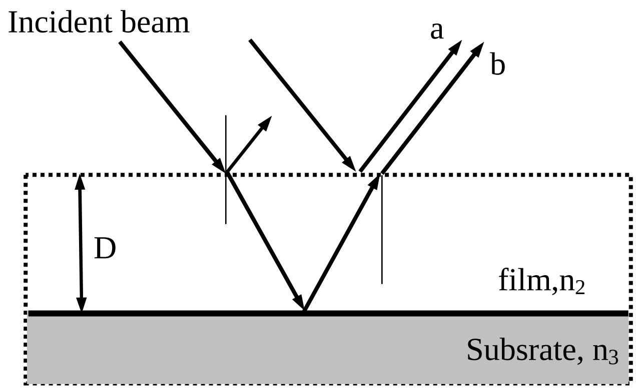


Figure 9 **:** *Reflection from first and second interfaces of a film[4]*

These two waves could interfere constructively if they meet two conditions. One condition is that the two waves must be relatively close together such that their crests and troughs can meet up with each other and cause the interference. A second condition which must be met is that the wave which travels through the film and back into the original medium must have travelled just the right distance such that it is *in phase* with the other reflected wave. Two waves which are in phase are waves which are always at the same point on their wave cycle. That is, the two waves must be forming crests at the same location and at the same moment in time and forming troughs at the same location and at the same moment in time. In order for the second condition to occur, the thickness of the film must be *just perfect*. If the material is thin means that thickness of the layer is very close to the wavelength of the light the possibility of second condition is very high. Also the same thing can occur for thick layers but it is not efficient way to consider thick materials with this way. Because there becomes no high effect on the results. Discussion on thickness and also relation between thickness and wavelength will be done in detail later. If *wave a* and *wave b* meet these two conditions as they reflect and exit the film, then they will constructively interfere. An important consideration in determining whether these waves interfere constructively or destructively is the fact that whenever light reflects off a surface of higher index of refraction, a 180° phase shift in the wave is introduced. In other words reflected waves from the film undergo a 180° phase shift when $n_2 > n_1$.

Light consists of a collection of light waves of varying wavelength. So for example a red light wave has a different wavelength than a orange light wave which has a different wavelength than a yellow light wave. While the thickness of a film at a given location may not allow a red and an orange light wave to emerge from the film in phase, it may be just perfect to allow a yellow light wave to emerge in phase. So at a given location on the film, the yellow light wave undergoes constructive interference and becomes brighter than the others within the incident light.

If the waves do not interfere than there becomes phase difference between two waves.

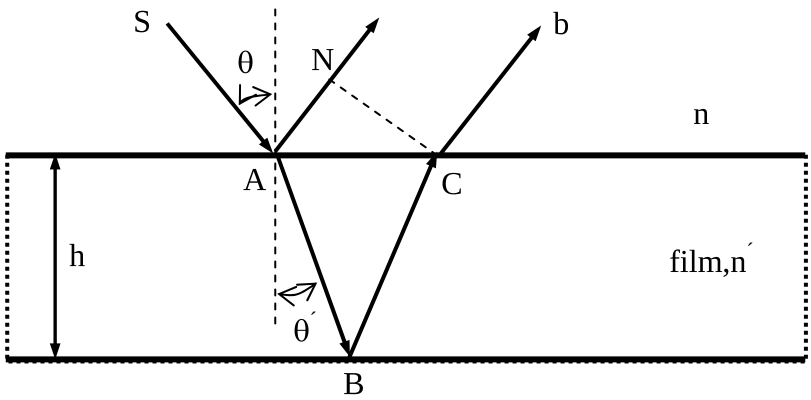


*Figure 10 **:** Phase difference between two waves[2]*

Beam b reflected from the second interface travels ΔS farther than beam a, which is reflected from the first interface(Figure 10).

$$\Delta S = n'(AB + BC) - nAN \tag{1.95}$$

n' is refractive index of surrounding, n is refractive index of the film and N is foot of the perpendicular from C to AD.

$$AB = BC = \frac{h}{\cos\theta'} \tag{1.96}$$

$$AC = 2h\tan\theta' \tag{1.97}$$

$$AN = AC\sin\theta \tag{1.98}$$

$$n'\sin\theta' = n\sin\theta \tag{1.99}$$

If Equations (1.97) and (1.98) are combined,

$$AN = 2h\tan\theta'\sin\theta \tag{1.100}$$

Then ΔS can be obtained by using Eq.(1.95),(1.96),(1.99) and (1.100),

$$\Delta S = 2n'h\cos\theta' \tag{1.101}$$

Reflected beam b originated at time $\Delta S / c_1$ earlier than reflected beam a, where c_1 is the propagation speed in the layer. If beam a originated at time 0, the beam b originated at time -$\Delta S / c_1$. If the two waves originated from the same vibrating source, the phase of b relative to a is,

$$e^{i\omega\tau} = e^{-i\omega\Delta S/c_1} \tag{1.102}$$

The circular frequency can be written as $\omega = 2\pi c_0 / \lambda_0$, where λ_0 is the wavelength in vacuum. And also $n_1 = c_0 / c_1$ is the film refractive index. Then *the phase* is,

$$e^{i\omega\tau} = e^{-i4\pi n_1 h\cos\theta'/\lambda_0} \tag{1.103}$$

And the phase difference is,

$$\gamma_1 = 4\pi n_1 h\cos\theta' / \lambda_0 \tag{1.104}$$

Now refraction and reflection phenomena is started to discuss. To investigate the surface reflection and refraction vector amplitudes which are the Fresnel's coefficients (i.e $E_{M\|,r} / E_{M\|,i}$), the electromagnetic wave equations is considered again. In section 1.3.1 the components of the plane electric and magnetic field of incident, refracted, and reflected waves (polarized in parallel and perpendicular to plane of incidence) in the x,y,z coordinate system are calculated. These equations will be directly used and will not be proofed again.(as an example see the Figure 11 for the components of the plane electric field incident, refracted, and reflected waves (polarized in parallel to plane of incidence) in the x,y,z coordinate system).

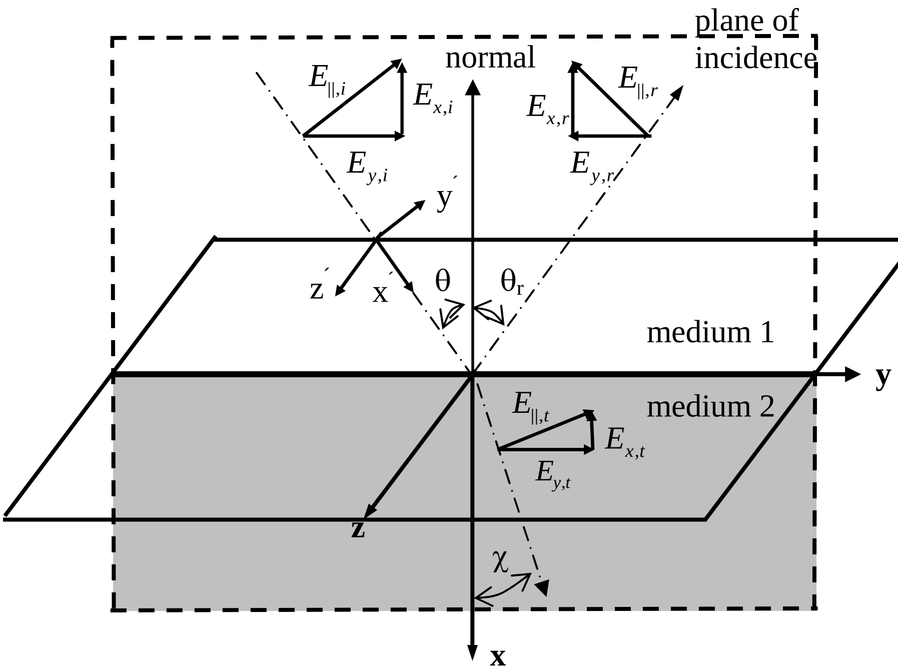


*Figure 11 **:** The components of the plane electric field (polarized in parallel to plane of incidence) incident, refracted, and reflected waves in the x,y,z coordinate system*

Equation (1.66) is the relation between components of the plane electric fields which is polarized in parallel to plane of incidence,

$$(E_{M\|,i}\cos\theta - E_{M\|,r}\cos\theta = E_{M\|,t}\cos\chi)_{x=0} \tag{1.105}$$

Equation (1.69) is the relation between components of the plane magnetic fields which is polarized in perpendicular to plane of incidence,

$$(n_1 E_{M\|,i} + n_1 E_{M\|,r} = n_2 E_{M\|,t})_{x=0} \tag{1.106}$$

Equation (1.71) is the relation between components of the plane electric fields which is polarized in perpendicular to plane of incidence,

$$E_{M\perp,i} + E_{M\perp,r} = E_{M\perp,t} \tag{1.107}$$

Equation (1.73) is the relation between components of the plane magnetic fields which is polarized in parallel to plane of incidence,

$$(E_{M\perp,i} n_1 \cos\theta - E_{M\perp,r} n_1 \cos\theta = E_{M\perp,t} n_2 \cos\chi)_{x=0} \tag{1.108}$$

For the representation of surface reflectance vector amplitude `r´; for the representation of surface transmission vector amplitude `t´ are used.

Because of possibility of the phase shift in the wave interference effect, surface reflectance and transmission vector amplitudes from medium 1 to medium 2 and medium 2 to medium 1 are different. The representation of reflection and transmission of the surface in both direction can be seen in Figure 12.

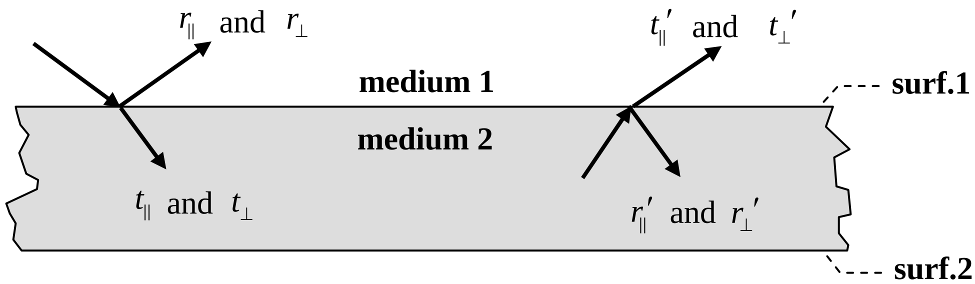


*Figure 12 **:** Surface reflectance and transmission vectors-Fresnel's coefficients*

Equations (1.105) and (1.106) are combined to eliminate $E_{M\|,t}$ and give the parallel reflection vector amplitude from medium 1 to medium 2(Figure 12) for non-attenuating materials as,

$$\frac{E_{M\|,r}}{E_{M\|,i}} = r_\| = \frac{\cos\theta / \cos\chi - n_1 / n_2}{\cos\theta / \cos\chi + n_1 / n_2} \tag{1.109}$$

Equations (1.107) and (1.108) are combined to eliminate $E_{M\perp,t}$ and give the perpendicular reflection vector amplitude from medium 1 to medium 2(Figure 12) for non-attenuating materials as,

$$\frac{E_{M\perp,r}}{E_{M\perp,i}} = r_\perp = -\frac{\cos\chi / \cos\theta - n_1 / n_2}{\cos\chi / \cos\theta + n_1 / n_2} \tag{1.110}$$

Equations (1.105) and (1.106) are combined to eliminate $E_{M\|,r}$ and give the parallel transmission vector amplitude from medium 1 to medium 2(Figure 12) for non-attenuating materials as,

$$\frac{E_{M\|,t}}{E_{M\|,i}} = t_\| = \frac{2 n_1 \cos\theta}{n_1 \cos\chi + n_2 \cos\theta} \tag{1.111}$$

Equations (1.107) and (1.108) are combined to eliminate $E_{M\perp,r}$ and give the perpendicular transmission vector amplitude from medium 1 to medium 2(Figure 12) for non-attenuating materials as,

$$\frac{E_{M\perp,r}}{E_{M\perp,i}} = t_{\perp} = \frac{2n_1 \cos\theta}{n_1 \cos\theta + n_2 \cos\chi} \tag{1.112}$$

In going from medium 2 to medium 1(Figure 12), the χ and θ values are interchanged in the above relations and the Fresnel's coefficients are,

$$r_{\parallel}' = -r_{\parallel} \qquad r_{\perp}' = -r_{\perp} \tag{1.113}$$

$$t_{\parallel}' = \frac{2n_2 \cos\chi}{n_2 \cos\theta + n_1 \cos\chi} \qquad t_{\perp}' = \frac{2n_2 \cos\chi}{n_2 \cos\chi + n_1 \cos\theta} \tag{1.114}$$

The Fresnel's coefficients are spectral(depends on wavelength) because n_1 and n_2 are functions of λ.[3]

Because the waves affect each other, during a layer computation or a group of layers computation the surface reflectance and transmission will not be a subject of discussion. The reflection and the transmission vector amplitudes will be used directly for the system. This topic will be explained in details later.

1.2.4 Fresnel Coefficients between Two Absorbing Medium ($\kappa \neq 0$) (With wave interference effect)

The propagation of a wave in an infinite medium that attenuates the wave is governed by the same relations as in a non-attenuating medium if the refractive index \`n´ is replaced by $\bar{n} = n - i\kappa$ [4]. When the interaction of a wave with a boundary is considered, the theoretical expressions for reflection vector amplitudes derived for non-absorbing media ($\kappa = 0$) also apply for attenuating media if $\bar{n}$ is used instead of n(as in Section 1.2.2).

The Snell's law(Eq.(1.81)) is explained in the Section 1.2.2.

For incident rays polarized parallel or perpendicular to the plane of incidence, Fresnel's coefficients(medium 1 to medium 2 (Figure 12)) are obtained,

$$\frac{E_{M\parallel,r}}{E_{M\parallel,i}} = r_{\parallel} = \frac{\cos\theta / \cos\chi - (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}{\cos\theta / \cos\chi + (n_1 - i\kappa_1)/(n_2 - i\kappa_2)} \tag{1.115}$$

$$\frac{E_{M\perp,r}}{E_{M\perp,i}} = r_{\perp} = -\frac{\cos\chi / \cos\theta - (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}{\cos\chi / \cos\theta + (n_1 - i\kappa_1)/(n_2 - i\kappa_2)} \tag{1.116}$$

$$\frac{E_{M\parallel,t}}{E_{M\parallel,i}} = t_{\parallel} = \frac{2(n_1 - i\kappa_1)\cos\theta}{(n_1 - i\kappa_1)\cos\chi + (n_2 - i\kappa_2)\cos\theta} \tag{1.117}$$

$$\frac{E_{M\perp,r}}{E_{M\perp,i}} = t_{\perp} = \frac{2(n_1 - i\kappa_1)\cos\theta}{(n_1 - i\kappa_1)\cos\theta + (n_2 - i\kappa_2)\cos\chi} \tag{1.118}$$

In going from medium 2 to medium 1(Figure 12), the χ and θ values are interchanged in the above relations,

$$r_{\parallel}' = -r_{\parallel} \qquad r_{\perp}' = -r_{\perp} \tag{1.119}$$

$$t_{\parallel}' = \frac{2(n_2 - i\kappa_2)\cos\chi}{(n_2 - i\kappa_2)\cos\theta + (n_1 - i\kappa_1)\cos\chi} \tag{1.120}$$

$$t_{\perp}' = \frac{2(n_2 - i\kappa_2)\cos\chi}{(n_2 - i\kappa_2)\cos\chi + (n_1 - i\kappa_1)\cos\theta} \tag{1.121}$$

The Fresnel's coefficients are spectral(depends on wavelength) because n_1, n_2, and κ are functions of λ.[3]

Index of refraction is complex number, so the above ratios are also complex. Since waves interfere each other, surface reflection and transmission values are not considered separately from the system and complex forms of above equations will be used in the computation of layers variables.

2 Transmission, Absorption and Reflection of Systems

The radiative behaviour of single and multilayered windows is very important for heating and cooling process. Multiple reflections from the window surfaces can be appreciable in reducing transmission, especially if more than one window layer is present. Transmission losses through multiple windows can be significant and relations for their calculation are required. Transient temperature distributions are important for glass and other translucent materials where there is internal absorption and emission of radiation in the heated materials, combined with heat conduction.

If one or more reflecting translucent layers are directly attached to an opaque surface or to other reflecting translucent layers, a coating system can be formed that has desirable radiative properties. A coating can be either thick or thin compared with the radiation wavelength(means wave interference effects are important or not). Thin films produce wave interference effects between incident and reflected waves in the film, thus influencing their reflection and transmission characteristics. A composite window can be formed that is selectively transmitting, or a coated surface that is selectively absorbing can be produced. Films that are thick relative to the radiation wavelength can also be used to modify surface radiative characteristics. Since windows and coatings are often thin temperature variations within them are not considered. However there are applications in which the temperature distributions must be obtained within a translucent medium, including the effects of interface reflections. This requires solving the transfer equations with in the medium with the addition of reflection boundary conditions at the interface. But this will not be a subject of our discussion.[4]

Enclosures can have windows that are partially transparent to radiation. A window can be of a single material, or it can have one or more transmitting coatings on it. The transparency is a function of the window and coating thicknesses and can be very wavelength dependent. This section is concerned with the transmission of incident radiation as influenced by surface reflections and absorptions within a layer without scattering.

2.1 Single Partially Transmitting Layer with Thickness D>λ(No Wave Interference Effect)

When a window or transmitting layer is very thin, so that its thickness is comparable to the radiation wavelength, there can be interference between incident and reflected waves. For the present discussion a window where D is at least several wavelengths thick so interference effects are usually lost.[4] To calculate the transmission, reflection, and absorption of the single layer system two methods is considered. These are Ray Tracing Method and Net Radiation Method.

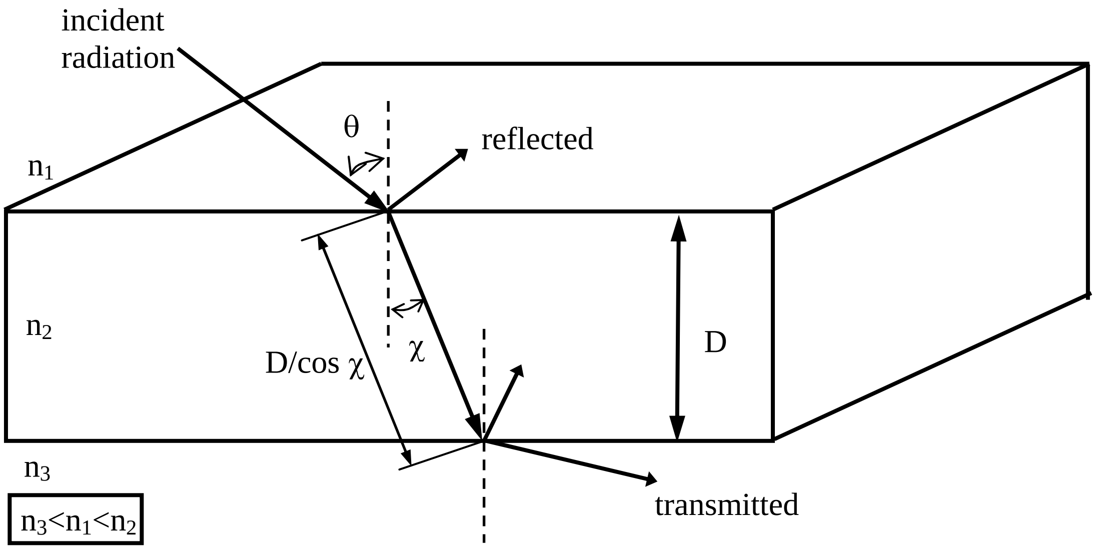


Figure 13 : Reflection and transmission of incident radiation by a partially transmitting layer

2.1.1 Ray Tracing Method

Before start to discuss the method in detail, mediums at the front and back side of the thick layer are decided.

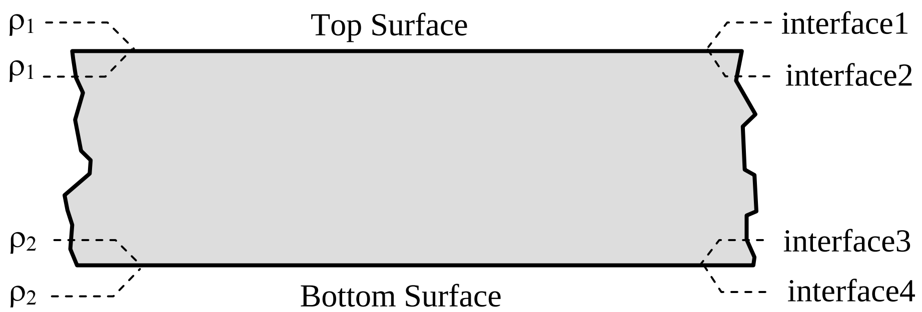


Figure 14 : Surface reflectance of a layer if front and back mediums are different

Assume the thickness of the layer is thick enough that there is no interference effect. For that reason the reflection and transmission of the interface 1 is same with interface 2; interface 3 is same with interface 4. In other words ρ_1 is the reflection of the top surface and ρ_2 is the reflection of the bottom surface. Apart from that the mediums which are front and back side of the layer are different. So the reflectivity and transmissivity of the top and bottom surfaces are different.

2.1.1.1 Non-attenuating (Perfect Dielectric) Materials

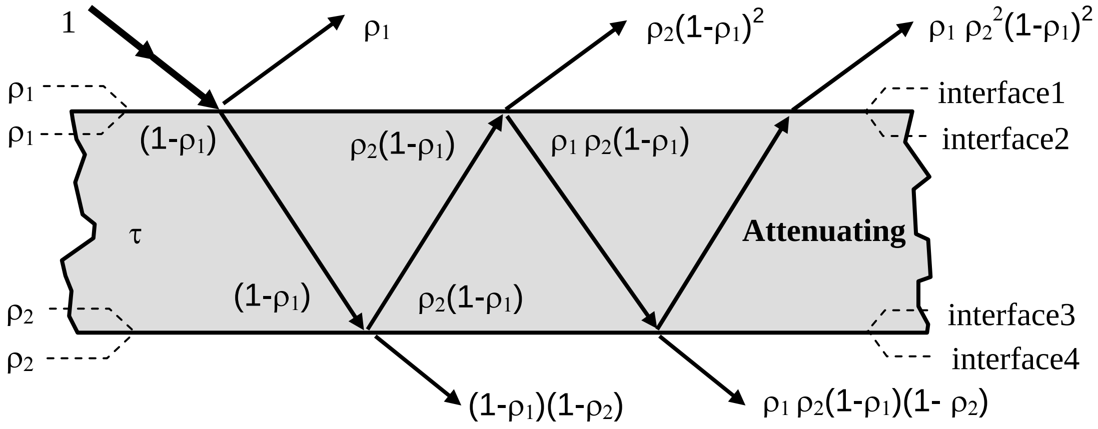


Figure 15 : Multiple internal reflections for non-attenuating materials

Incident wave with a unity intensity is considered on the upper boundary(Figure 15). At the contact with the interface 1, an amount ρ_1 is reflected so that $(1-\rho_1)$ enter the material. Because of non-absorbing material property, all the intensity which go into the material reach to the interface 3 without any absorption losses. Of this, $(1-\rho_1)\rho_2$ is reflected from interface 3 and the $(1-\rho_1)(1-\rho_2)$ pass out of the layer through the lower boundary.

As the process continues, the fraction of incident energy reflected by the layer is the sum of the terms leaving top surface:

$$R = \rho_1 + (1-\rho_1^2)(\rho_2 + \rho_1\rho_2^2 + \rho_1^2\rho_2^3 + ...)] = \frac{\rho_1 + \rho_2(1-2\rho_1)}{1-\rho_1\rho_2}] \tag{2.1}$$

The fraction of incident energy transmitted by the layer is the sum of the terms leaving bottom surface:

$$T = (1-\rho_1)(1-\rho_2)[1 + \rho_1\rho_2 + \rho_1^2\rho_2^2 + ...] = \frac{(1-\rho_1)(1-\rho_2)}{1-\rho_1\rho_2} \tag{2.2}$$

Because there is no absorbing losses the fraction of incident energy absorbed by the layer,

$$A = 0 \tag{2.3}$$

All three coefficients only depend on the surface reflectance. In other words, the thickness of the non-attenuating material has no affect on the transmission and reflection if there is no wave interference effect.

If the back and front environments of the layer is same then top surface reflectance and transmission is the same with the bottom surface which means that $\rho_1 = \rho_2 = \rho$. And for that special case the above equations turn into form of,

$$R = \rho[1 + \frac{(1-\rho)^2}{1-\rho^2}] = \frac{2\rho}{1+\rho} \qquad T = \frac{(1-\rho)^2}{1-\rho^2} = \frac{1-\rho}{1+\rho} \qquad A = 0 \tag{2.4}$$

2.1.1.2 Attenuating Materials

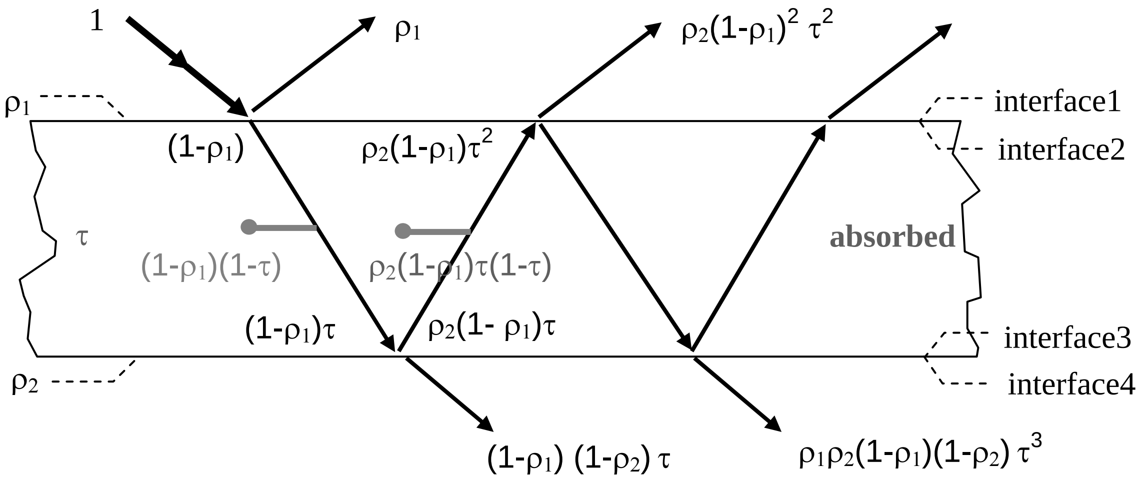


*Figure 16 **:** Multiple internal reflections for attenuating materials*

Incident wave with a unity intensity is considered on the upper boundary(see Figure 16). At the contact with the interface 1, an amount ρ_1 is reflected so that $(1-\rho_1)$ enter the material. Of this, $(1-\rho_1)\tau$ is transmitted to the interface 3 and $(1-\rho_1)(1-\tau)$ is absorbed along the path. At the interface 3, $\rho_2(1-\rho_1)\tau$ is reflected in to the layer and $(1-\rho_2)(1-\rho_1)\tau$ pass out of the layer through the lower boundary.

As the process continues, the fraction of incident energy reflected by the layer is the sum of the terms leaving top surface:

$$R = \rho_1 + (1-\rho_1^2)\tau^2(\rho_2 + \rho_1\rho_2^2\tau^2 + \rho_1^2\rho_2^3\tau^4 + ...)] = \frac{\rho_1 + \rho_2(1-2\rho_1)\tau^2}{1-\rho_1\rho_2\tau^2}] \tag{2.5}$$

The fraction of incident energy transmitted by the layer is the sum of the terms leaving bottom surface:

$$T = (1-\rho_1)(1-\rho_2)\tau[1 + \rho_1\rho_2\tau^2 + \rho_1^2\rho_2^2\tau^4 + ...] = \frac{(1-\rho_1)(1-\rho_2)\tau}{1-\rho_1\rho_2\tau^2} \tag{2.6}$$

The incident wave intensity is unity at the starting point so at the end if the fraction of incident energy transmitted, reflected and absorbed is added again unity is obtained. For that reason, the fraction of incident energy absorbed in the layer,

$$A = 1 - R - T \tag{2.7}$$

At that point the absorption losses can be neglected which means that $\tau=1$ and the Eq.(2.5), (2.6),and (2.7) turn into the form of,

$$R = \frac{\rho_1 + \rho_2(1-2\rho_1)}{1-\rho_1\rho_2}] \qquad T = \frac{(1-\rho_1)(1-\rho_2)}{1-\rho_1\rho_2} \qquad A = 0 \tag{2.8}$$

These are the equations for non-attenuating materials which are derived in the Section 2.1.1.1(see Eq.(2.1),(2.2),(2.3)). As a result, if τ value is 1 in the equations for attenuating materials then the equations for the non-absorbing materials are obtained.

2.1.2 Net radiation Method

The net radiation method is a powerful analytical tool. In many situations, it is much less difficult to apply than the ray tracing method. Now the configuration which is considered in the ray tracing method will be discussed with the net radiation method.[4]

Mediums at the front and back side of the thick layer are same as in the ray tracing method(Figure 12).

2.1.2.1 Non-Attenuating Materials

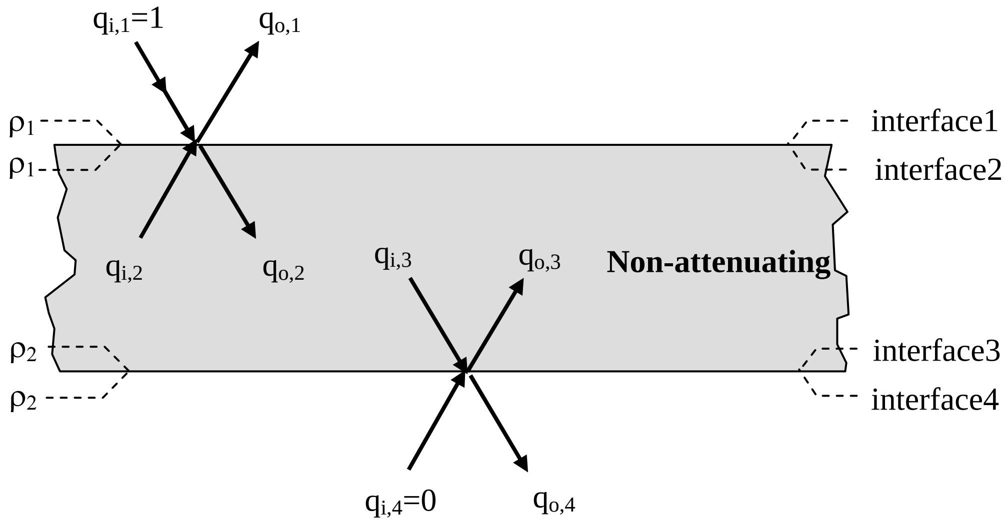


Figure 17 : Net radiation method applied to partially transmitting non-attenuating layer

It can be easily seen that from Figure 17 for interfaces there are some outgoing and ingoing fluxes. And outgoing fluxes at each interface can be written in terms of the incoming fluxes. Energy balance for the interface 1 is explained in detail as an example. The others are written directly.

For the interface 1, outgoing flux is $q_{o,1}$ is composed of incoming fluxes, $q_{i,2}$ and $q_{i,1}$. To create $q_{o,1}$, $q_{i,2}$ is transmitted from the layer through the front medium($(1-\rho_1)q_{i,2}$) and $q_{i,1}$ is reflected from the top surface($\rho_1 q_{i,1}$).As a result the energy balance for the interface 1,

$$q_{0,1} = \rho_1 q_{i,1} + (1-\rho_1) q_{i,2} = \rho_1 + (1-\rho_1) q_{i,2} \tag{2.9}$$

Each interface can be written in terms of the incoming fluxes to yield the following equations for the conditions of a unit incoming flux from a single direction at surface 1 and a zero incoming flux at surface 4 which means that there is no radiation form inside to outside. And the other energy balance equations are,

$$q_{0,2} = (1-\rho_1) q_{i,1} + \rho_1 q_{i,2} = (1-\rho_1) + \rho_1 q_{i,2} \tag{2.10}$$

$$q_{0,3} = \rho_2 q_{i,3} + (1-\rho_2) q_{i,4} = \rho_2 q_{i,3} \tag{2.11}$$

$$q_{0,4} = (1-\rho_2) q_{i,3} + \rho_2 q_{i,4} = (1-\rho_2) q_{i,3} \tag{2.12}$$

At that point, 4 equations are obtained from the energy balance. The surface reflectance,, and surface transmission, which are proofed by using electromagnetic wave theory, are known.

So the only unknowns are the outgoing and incoming fluxes. Before it is explained that $q_{i,1}$ is assumed as unity, and $q_{i,4}$ is assumed as zero. And the remaining 6 unknowns are $q_{o,1}$, $q_{i,2}$, $q_{o,2}$, $q_{i,3}$, $q_{o,3}$, $q_{o,3}$. To solve the above equations system, the last thing that have be considered is non-attenuating material property. Since there is no any energy loss because of absorption all the wave intensity is transmitted from top surface to the bottom surface and vice versa. As a results,

$$q_{0,3} = q_{i,2} \tag{2.13}$$

$$q_{0,2} = q_{i,3} \tag{2.14}$$

Now, 6 equations and 6 unknowns are obtained. If they are solved,

$$q_{0,1} = \frac{\rho_1 + \rho_2(1-2\rho_1)^2}{1-\rho_1\rho_2} \qquad q_{0,4} = \frac{(1-\rho_1)(1-\rho_2)}{1-\rho_1\rho_2} \tag{2.15}$$

The fractions reflected, transmitted and absorbed by the plate are,

$$R = q_{0,1} = \frac{\rho_1 + \rho_2(1-2\rho_1)^2}{1-\rho_1\rho_2} \tag{2.16}$$

$$T = q_{0,4} = \frac{(1-\rho_1)(1-\rho_2)}{1-\rho_1\rho_2} \tag{2.17}$$

$$A = 0 \tag{2.18}$$

These results agree, as they have to, with those obtained by the ray tracing method(see Eq.(2.1),(2.2), and (2.3)).

2.1.2.2 Attenuating Materials

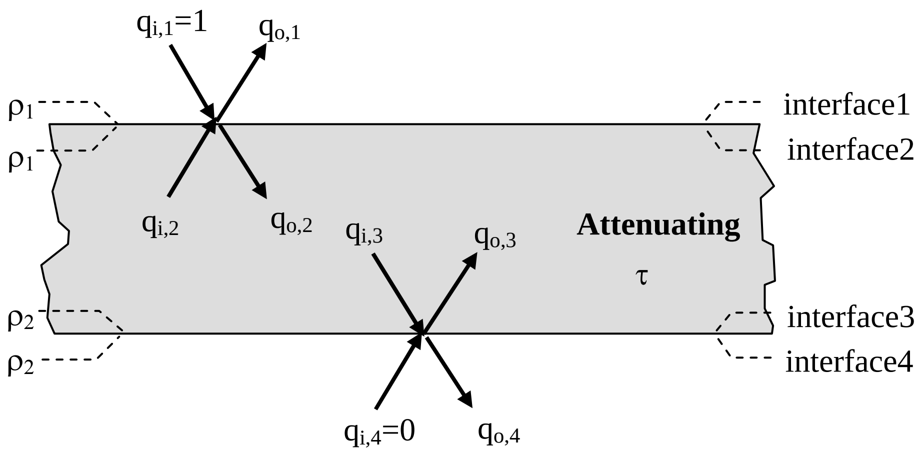


Figure 18 : Net radiation method applied to partially transmitting attenuating layer

Like in the section 2.1.2.1, each interface can be written in terms of the incoming fluxes to yield the following equations for the conditions of a unit incoming flux from a single direction at surface 1 and a zero incoming flux at surface 4 which means that there is no radiation form inside to outside. And the energy balance equations are,

$$q_{0,1} = \rho_1 q_{i,1} + (1-\rho_1)q_{i,2} = \rho_1 + (1-\rho_1)q_{i,2} \tag{2.19}$$

$$q_{0,2} = (1-\rho_1)q_{i,1} + \rho_1 q_{i,2} = (1-\rho_1) + \rho_1 q_{i,2} \tag{2.20}$$

$$q_{0,3} = \rho_2 q_{i,3} + (1-\rho_2) q_{i,4} = \rho_2 q_{i,3} \quad (2.21)$$

$$q_{0,4} = (1-\rho_2) q_{i,3} + \rho_2 q_{i,4} = (1-\rho_2) q_{i,3} \quad (2.22)$$

To solve the above equations system, attenuating material property is considered in the layer. Since there is energy loss because of absorption the transmittance of the layer is used to relate the internal fluxes to give,

$$q_{i,2} = q_{o,3}\tau \qquad q_{i,3} = q_{o,2}\tau \quad (2.23)$$

Now, 6 equations and 6 unknowns are obtained. If they are solved,

$$q_{0,1} = \frac{\rho_1 + \rho_2 (1-2\rho_1)^2 \tau^2}{1-\rho_1\rho_2\tau^2}] \qquad q_{0,4} = \frac{(1-\rho_1)(1-\rho_2)\tau}{1-\rho_1\rho_2\tau^2} \quad (2.24)$$

The fractions reflected, transmitted and absorbed by the plate are,

$$R = q_{0,1} = \frac{\rho_1 + \rho_2 (1-2\rho_1)^2 \tau^2}{1-\rho_1\rho_2\tau^2}] \quad (2.25)$$

$$T = q_{0,4} = \frac{(1-\rho_1)(1-\rho_2)\tau}{1-\rho_1\rho_2\tau^2} \quad (2.26)$$

$$A = 1 - T - R \quad (2.27)$$

These results agree with those obtained by the ray tracing method(see Eq.(2.5),(2.6),(2.7)).

And also if τ value is 1 in the equations for attenuating materials then the equations for the non-attenuating materials are obtained.

2.1.3 Averaged Properties

The solar radiation properties for the system discussed above are all implicit functions of wavelength, incident angle and polarization.[5]

2.1.3.1 Spectral average of the system reflection, transmission and absorption

The optical indices are not dependent on either angle of incidence or polarization, although they may be strongly dependent on wavelength. Having calculated the properties at all desired wavelengths, average properties for the glazing system can then be calculated using the general equation:

$$P = \frac{\int_a^b p(\lambda)\Phi_{solar}(\lambda)\Gamma_{solar}(\lambda)d\lambda}{\int_a^b \Phi_{solar}(\lambda)\Gamma_{solar}(\lambda)d\lambda} \quad (2.28)$$

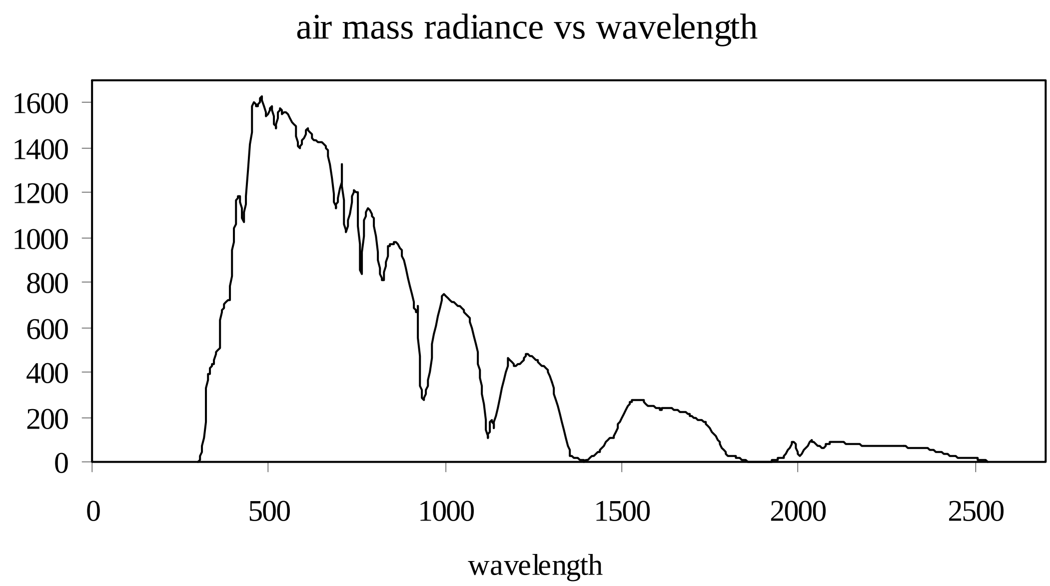
In the above equation, *p* is the calculated spectral radiometric property such as transmittance or reflectance; Φ is the weighting function for the strength of source (sun radiation) under appropriate conditions at each wavelength; and Γ is a weighting function for the response of the "detector."(Table 2)[5]The practical wavelength limits are specified by *a* and *b,* although the finite extent of the source or response functions effectively bound the integral.[5]

Property type	Property (p)	Lower wave.(a)	Upper wave.(b)	Source function (Φ)	Detector function (Γ)
Solar	T,R,A	0.30μm	2.535μm	AM 1.5 global Irradiance [ISO 9845/ASM 892]	1.0

T: transmission R: reflection A: absorption

Table 2 **:** *Values for spectral average[5]*

AM 1.5 global irradiance [ISO 9845/ASM 892] provides an appropriate standard spectral irradiance distribution to be used in determining relative performance of solar thermal, photovoltaic, and other system components and materials where the direct and hemispherical irradiance component is desired.[5] The tables are intended to represent ideal clear sky conditions. And the values are in the Appendix. Below the graph of the air mass solar spectral irradiance vs. wavelength can be found.



Graph 1 **:** *Air mass 1.5 terrestrial solar global spectral irradiance (W/m2-micron) on a 37 tilted surface*

At that point the two problematic parts are multiplication of the two functions which are p(λ) and Φ(λ), and integration process of this multiplication.

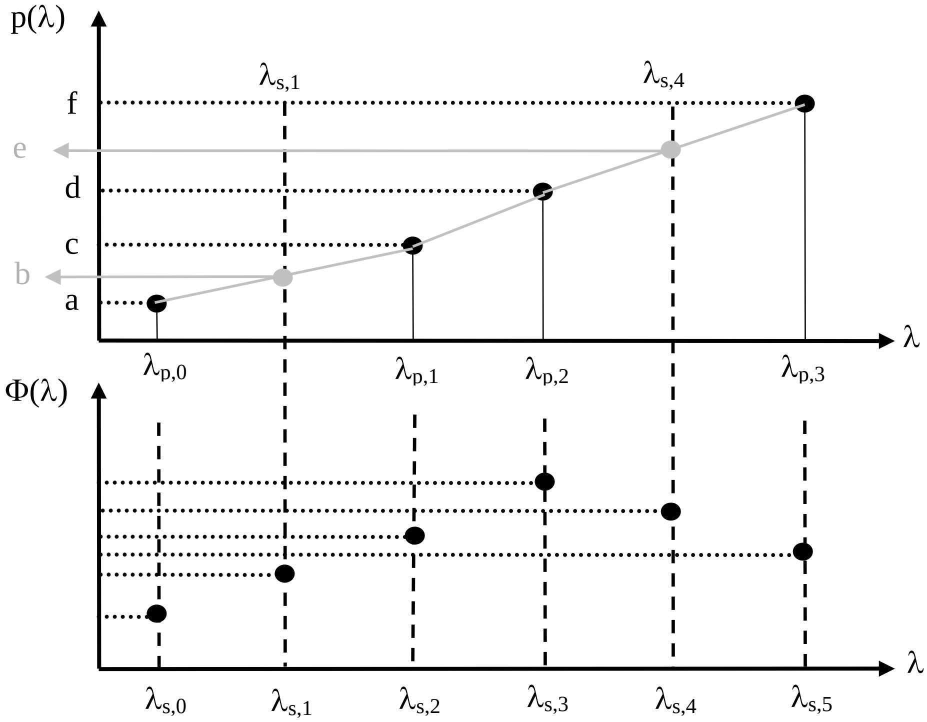


*Figure 19 **:** Master-slave functions and linear interpolation*

Why the multiplication causes a problem is wavelength increment of the two function. There is high probability of not matching of the all wavelengths like in the Figure 19. To solve this problem one of the two function is selected as a master and the other is slave. Here Φ(λ) is the master function. Because in the literature air mass solar radiation values are given for the more dense spectral values with respect to the refractive index and extinction coefficient values which determine the wavelength range of the calculated spectral radiometric property such as transmittance or reflectance(p(λ)). And also in the solar spectrum of air mass radiation the wavelength range is divided into equal parts. So in the slave function which is p(λ) linear interpolation is done for the not matching wavelengths. For example in the Figure 19 at two points the linear interpolation is needed,

$$b = a + \frac{\lambda_{s,1} - \lambda_{p,0}}{\lambda_{p,1} - \lambda_{p,0}}(c - a) \qquad e = d + \frac{\lambda_{s,4} - \lambda_{p,2}}{\lambda_{p,3} - \lambda_{p,2}}(f - d) \tag{2.29}$$

Finally the below graph is obtained for the p(λ),

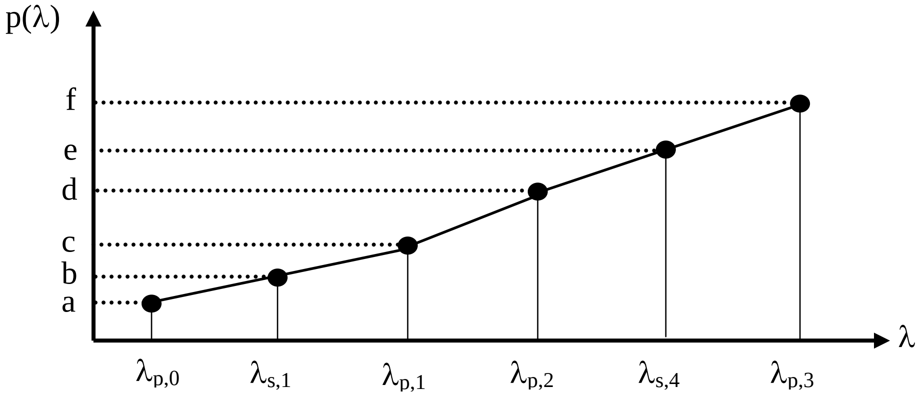


*Figure 20 **:** Linear interpolated spectral radiometric property such as transmittance or reflectance(p(λ))*

Now two function can be multiply without any doubt. But still one thing that is integration operation must be explained. The Newton cotes formulas are an extremely useful and straightforward family of numerical integration techniques. And they are suitable for this case because wavelength increment is always same which means that the wavelength interval is divided equally. To integrate a function $f(\lambda)$ over some interval $[a,b]$, divide it into n equal parts such that $f_n = f(\lambda_n)$ and $h=(b-a)/n$. Then find polynomials which approximate the tabulated function, and integrate them to approximate the area under the curve. To find the fitting polynomials, use Lagrange interpolating polynomials. Here the 2-point Newton-Cotes formula called the trapezoidal rule is used. It approximates the area under a curve by a trapezoid with horizontal base and sloped top (connecting the endpoints λ_1 and λ_2). The reason of choosing 2 point method is very small wavelength increment. So even the procedure is the simplest the error in the integration is acceptable.

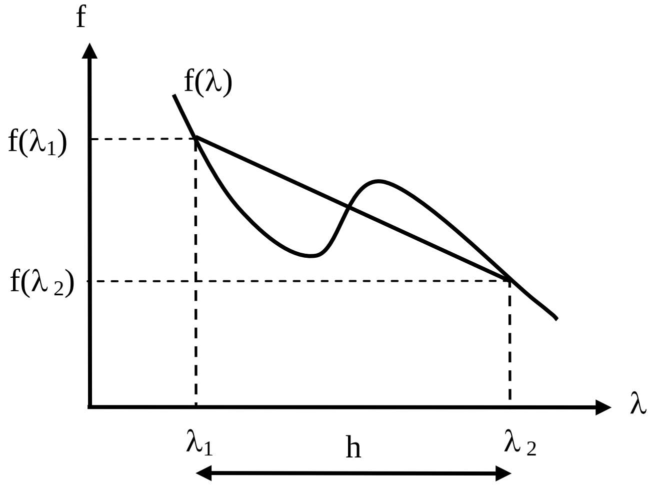


*Figure 21 **:** The 2-point Newton-Cotes formula is called the trapezoidal rule*

$$\int_a^b f(\lambda)d\lambda = \frac{1}{2}h[f(\lambda_1)+f(\lambda_2)] \quad (2.30)$$

2.1.3.2 Polarization average of the system reflection, transmission and absorption

For wave incidence on a material, polarization effects must be considered. Natural sunlight is “unpolarized” so that the angle of polarization with respect to the plane of incidence fluctuates randomly[5][1]. Only the time average is measurable for any kind of property *P* which reduces to a simple average of the properties for the two polarization states:

$$P = \frac{P_{\parallel} + P_{\perp}}{2} \quad (2.31)$$

2.1.4 Special Case

Up to know to calculate the transmission, reflection, and the absorption what are needed are extinction coefficient(κ), refractive index(n) and wavelengths(λ). But it is very hard to obtain those values for every material. Especially because extinction coefficient is very small and it is assumed zero in most cases for glazing materials , it cannot be found very easily in the literature. On the other hand, it is possible to obtain the experimental average normal transmission value from the glazing industry. So if this is the case, by using the average transmission value average absorption coefficient(a) can be calculated. The experiment for the transmission is done generally under the same front and back side mediums and it is air so $\rho_1 = \rho_2 = \rho$. As a result the surface reflection(Eq.(1.87)) and layer transmission(Eq.(2.26)) in these conditions and at normal incidence are,

$$\rho = \frac{(n_{layer} - 1)^2 + (\kappa_{layer} - 1)^2}{(n_{layer} + 1)^2 + (\kappa_{layer} + 1)^2} \qquad T = \frac{(1-\rho)^2 \tau}{1 - \rho^2 \tau^2} \tag{2.32}$$

But for the glazing materials extinction coefficients(κ_{layer}) is very close to zero and it can be neglected in the surface reflection formula. The transmission under the absorption losses for the normal incidence is (Eq.(1.49)),

$$\tau = \exp(ax) \tag{2.33}$$

If the layer thickness(x) , the refractive index(n_{layer}), and averaged normal incidence transmission(T) of the film are known than the averaged absorption coefficient(a) is calculated by combining the above formulas. After that layer solar radiation coefficients are obtained at different incidence angles by using the appropriate equations.

2.1.5 Some Examples

Example 1:

What is the transmission of the non-attenuating window in the air,
a) for the incident angle 50 degree?
b) for the incident angle between 0-90 with the increment of 1 degree?
(The window is 0.75cm thick, n=1.53. Assume that average value of refractive index is given.)

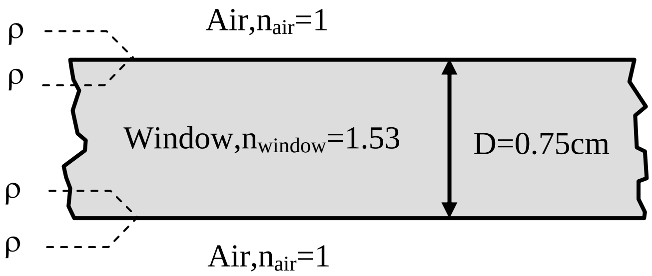


*Figure 22 **:** Properties of layer for* Example 1

$$T = \frac{(1-\rho)(1-\rho)}{1-\rho\rho} \qquad R = \frac{\rho + \rho(1-2\rho)^2}{1-\rho\rho}$$

The system coefficients for non-attenuating materials depend on surface reflectance(ρ), and surface transmission(1-ρ). The equations below are taken from the section 1,

$$\rho_{\parallel} = \left(\frac{\cos\theta / \cos\chi - n_1 / n_2}{\cos\theta / \cos\chi + n_1 / n_2}\right)^2 \qquad \rho_{\perp} = \left(-\frac{\cos\chi / \cos\theta - n_1 / n_2}{\cos\chi / \cos\theta + n_1 / n_2}\right)^2$$

Here the surface reflectance and also system transmission and reflection are not spectral because of average value of refractive index

a) $\chi = \sin^{-1}(\frac{n_{air} \sin\theta}{n_{window}}) = \sin^{-1}(\frac{\sin 50}{1.53}) = 30°$ (Snell's law)

$$\rho_{\parallel} = (\frac{\cos 50 / \cos 30 - 1/1.53}{\cos 50 / \cos 30 + 1/1.53})^2 = 0.00412 \quad \text{(parallel component)}$$

$$\rho_{\perp} = (-\frac{\cos 30 / \cos 50 - 1/1.53}{\cos 30 / \cos 50 + 1/1.53})^2 = 0.1206 \quad \text{(perpendicular component)}$$

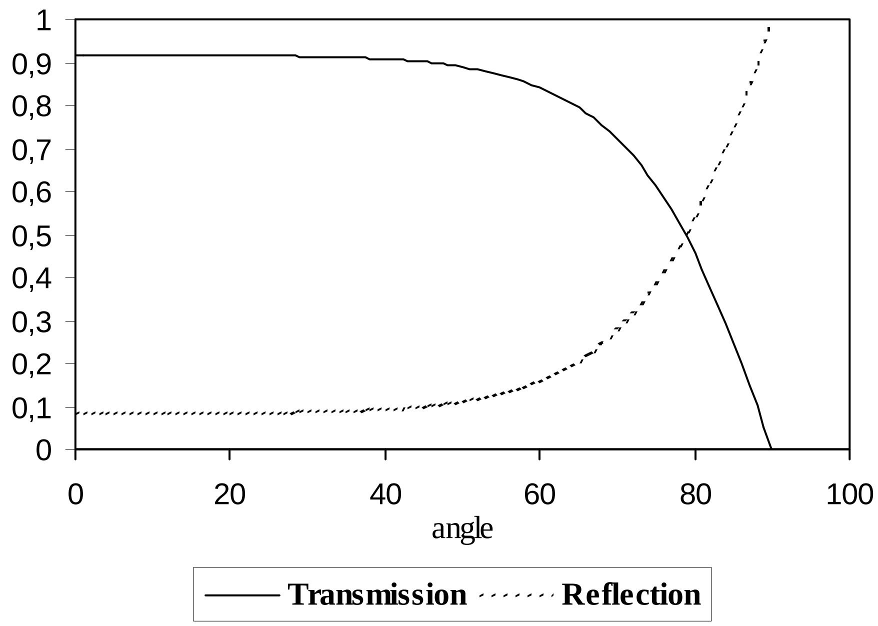
$$T_{\parallel} = \frac{(1-\rho_{\parallel})(1-\rho_{\parallel})}{1-\rho_{\parallel}\rho_{\parallel}} = 0.991794 \qquad T_{\perp} = \frac{(1-\rho_{\perp})(1-\rho_{\perp})}{1-\rho_{\perp}\rho_{\perp}} = 0.784758$$

For unpolarized incident radiation, one-half the energy is in each component. Hence,

$$T = \frac{T_{\parallel} + T_{\perp}}{2} = 0.888$$

It can be very easily seen that if there is no wave interference effect in the non-attenuating material than reflectance and transmission do not depend on the thickness of the layer.

b) This part is solved by using the software tool. And the solution is in the excel sheet which can be found very easily in the CD-Excel sheets-Example 1. The procedure that is followed in the previous question is applied for every angle. The final results is,



*Graph 2 **:** Transmission, Reflection values for angle of incidence between 1-90 degree*

Example 2:

What is the transmission of the attenuating window in the air,
a) for the incident angle 50 degree?
b) for the incident angle between 0-90 with the increment of 1 degree?
(The window is 0.75cm thick, n=1.53, κ= 4.85×10^{-7} Assume that average value of refractive index and extinction coefficient are given and average value of the wavelength is $619.02\times10^{-7}\,cm$)

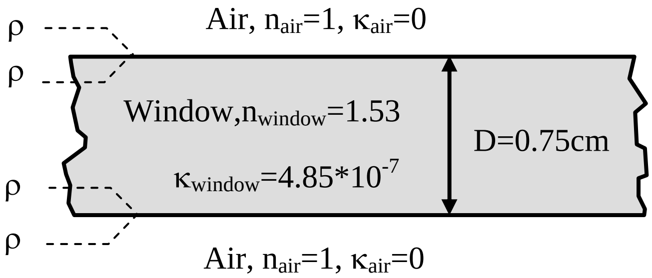


*Figure 23 **:** Properties of layer for Example 2*

Here the surface reflectance and also system transmission and reflection are not spectral because of average value of refractive index and extinction coefficient.

a) $$\sin\chi = \frac{(n_{air} - i\kappa_{air})\sin\theta}{n_{window} - i\kappa_{window}} = \frac{\sin 50}{1.53 - i4.85\times10^{-7}} = 0.5 + i1.59\times10^{-7} \quad \text{(Fresnel's law)}$$

$$\cos\chi = \sqrt{1-\sin^2\chi} = \sqrt{1-(0.5+i1.59\times10^{-7})^2} = 0.865 - i9.186\times10^{-8}$$

The surface reflectance,

$$\frac{E_{M\|,r}}{E_{M\|,i}} = \frac{\cos\theta/\cos\chi - (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}{\cos\theta/\cos\chi + (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}$$

$$= \frac{\cos 50/(0.865 - i9.186\times10^{-8}) - (1)/(1.53 - i4.85\times10^{-7})}{\cos 50/(0.865 - i9.186\times10^{-8}) + (1)/(1.53 - i4.85\times10^{-7})}$$

$$= \frac{(0.743 + i7.89\times10^{-8}) - (0.654 + 20.72\times10^{-8})}{(0.743 + i7.89\times10^{-8}) + (0.654 + 20.72\times10^{-8})} = 0.0637 - i1.049\times10^{-7}$$

$$\frac{E_{M\perp,r}}{E_{M\perp,i}} = -\frac{\cos\chi/\cos\theta - (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}{\cos\chi/\cos\theta + (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}$$

$$= -\frac{(0.865 - i9.186\times10^{-8})/\cos 50 - (1)/(1.53 - i4.85\times10^{-7})}{(0.865 - i9.186\times10^{-8})/\cos 50 + (1)/(1.53 - i4.85\times10^{-7})}$$

$$= -\frac{(1.345 - i1.59\times10^{-7}) - (0.654 + 2.072\times10^{-7})}{(1.345 - i1.59\times10^{-7}) + (0.654 + 2.072\times10^{-7})} = -(0.346 - i1.92\times10^{-7})$$

$$\rho_{\|}(\theta) = \left(\frac{E_{M\|,r}}{E_{M\|,i}}\right)^2 = (0.0637 - i1.049\times10^{-7})\times(0.0637 + i1.049\times10^{-7}) = 0.0041$$

$$\rho_{\perp}(\theta) = \left(\frac{E_{M\perp,r}}{E_{M\perp,i}}\right)^2 = (0.346 - i1.92\times10^{-7})\times(0.346 + i1.92\times10^{-7}) = 0.12$$

Transmission in the layer under the absorption losses,

$$\tau = \exp(-4\pi\kappa \frac{x}{\lambda_{a\,\mathrm{var}\,age} \cos\chi})$$

$$= \exp(-4\pi 4.85\times10^{-7} \frac{0.75}{619.02\times10^{-7}\times(0.865 - i9.186\times10^{-8})}) = 0.917$$

complex term is omitted(because very close to zero)

Layer transmission is,

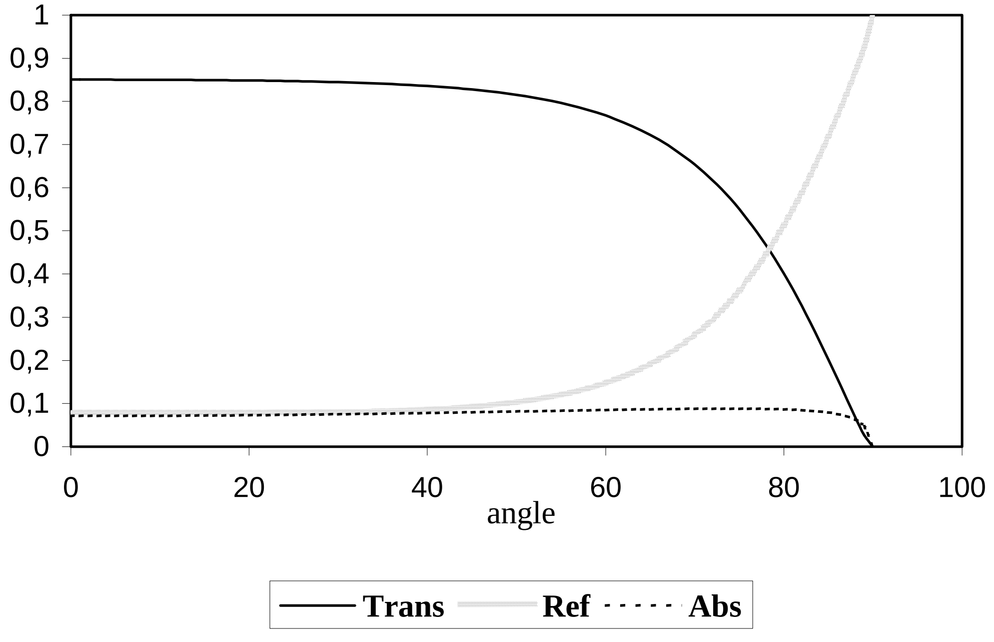
$$T_{\|} = \frac{(1-\rho_{\|})(1-\rho_{\|})\tau}{1-\rho_{\|}\rho_{\|}\tau^2} = \frac{(1-0.0041)(1-0.0041)0.917}{1-0.0041\times0.0041\times0.917\times0.917} = 0.91$$

$$T_{\perp} = \frac{(1-\rho_{\perp})(1-\rho_{\perp})\tau}{1-\rho_{\perp}\rho_{\perp}\tau^2} = \frac{(1-0.12)(1-0.12)0.917}{1-0.12\times0.12\times0.917\times0.917} = 0.719$$

For unpolarized incident radiation, one-half the energy is in each component. Hence,

$$T = \frac{T_{\|}+T_{\perp}}{2} = \frac{0.91+0.719}{2} = 0.82$$

b) This part is solved by using the software tool. And the solution is in the excel sheet which can be found very easily in the CD-Excel sheets-Example 2. The procedure that is followed in the previous question is applied for every angle. The final results is,



*Graph 3 **:** Transmission, Reflection and Absorption values for angle of incidence between 1-90 degree*

Example 3:

What is the transmission of the attenuating window in the air for the incident angle 50 degree?(The window is 0.075cm thick.)

Wavelength dependent refractive index and extinction coefficient for the window material are,

Wavelength	Refractive Index	Extinction Coefficient
0.000061902	3.9561	0.00007749
0.000062722	3.94	7.39888E-05
0.000063542	3.9245	7.06782E-05
0.000064362	3.9095	0.000067545
0.000065182	3.895	6.45736E-05
0.000066002	3.881	6.17497E-05
0.000066822	3.8675	5.90611E-05
0.000067642	3.8544	5.64965E-05
0.000068462	3.8418	5.40459E-05
0.000069282	3.8295	5.17005E-05

*Table 3 **:** Refractive index and extinction coefficient for the* Example 3

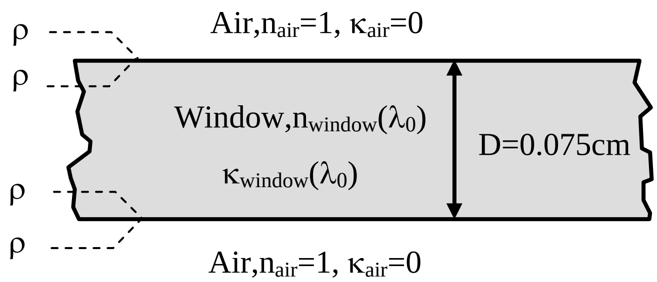


*Figure 24 **:** Properties of layer for* Example 3

For the wavelength λ_0 =0.000061902 and n=3.9561 and κ=0.00007749:

$$\sin\chi = \frac{(n_{air} - i\kappa_{air})\sin\theta}{n_{window} - i\kappa_{window}} = \frac{\sin 50}{3.9561 - i7.749\times10^{-5}} = 0.194 + i3.79\times10^{-6}$$

$$\cos\chi = \sqrt{1-\sin^2\chi} = \sqrt{1-(0.194+i3.79\times10^{-6})^2} = 0.981 - i7.49\times10^{-7}$$

The surface reflectance,

$$\rho_{\|} = (\frac{E_{M\|,r}}{E_{M\|,i}})^2 = (\frac{\cos\theta/\cos\chi - (n_1 - i\kappa_1)/(n_2 - i\kappa_2)}{\cos\theta/\cos\chi + (n_1 - i\kappa_1)/(n_2 - i\kappa_2)})^2$$

$$= (\frac{\cos 50/\cos\chi - (1)/(3.9561 - i0.00007749)}{\cos 50/\cos\chi + (1)/(3.9561 - i0.00007749)})^2 = 0.196$$

$$\rho_\perp = \left(\frac{E_{M\perp,r}}{E_{M\perp,i}}\right)^2 = \left(-\frac{\cos\chi/\cos\theta-(n_1-i\kappa_1)/(n_2-i\kappa_2)}{\cos\chi/\cos\theta+(n_1-i\kappa_1)/(n_2-i\kappa_2)}\right)^2$$
$$= \left(-\frac{\cos\chi/\cos 50-(1)/(3.9561-i0.00007749)}{\cos\chi/\cos 50+(1)/(3.9561-i0.00007749)}\right)^2 = 0.512$$

Transmission in the layer under the absorption losses,

$$\tau = \exp(-4\pi\kappa\frac{x}{\lambda_0}) = \exp(-4\pi 0.00007749\frac{0.075}{0.000061902\cos\chi}) = 0.3$$

Layer transmission is,

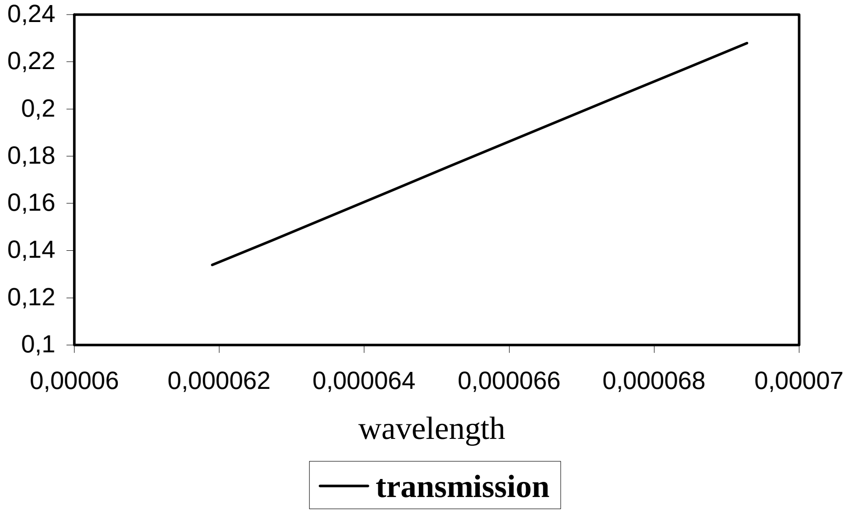
$$T_\parallel = \frac{(1-\rho_\parallel)(1-\rho_\parallel)\tau}{1-\rho_\parallel\rho_\parallel\tau^2} = \frac{(1-0.196)(1-0.196)0.3}{1-0.196\times0.196\times0.3\times0.3} = 0.195$$
$$T_\perp = \frac{(1-\rho_\perp)(1-\rho_\perp)\tau}{1-\rho_\perp\rho_\perp\tau^2} = \frac{(1-0.512)(1-0.512)0.3}{1-0.512\times0.512\times0.3\times0.3} = 0.0732$$

For unpolarized incident radiation, one-half the energy is in each component. Hence,

$$T = \frac{T_\parallel+T_\perp}{2} = \frac{0.195+0.0732}{2} = 0.134$$

If this procedure is applied to other wavelengths separately the below table is obtained(by using software tool),

Wavelength:	**Transmission:**	**Wavelength:**	**Transmission:**
6.1902e-005	0.133911	6.6002e-005	0.186226
6.2722e-005	0.144286	6.6822e-005	0.196706
6.3542e-005	0.154733	6.7642e-005	0.207153
6.4362e-005	0.16522	6.8462e-005	0.217549
6.5182e-005	0.175725	6.9282e-005	0.227891



*Graph 4 **:** Transmission vs. Wavelength for attenuating layer in* Example 3

At that point transmission is integrated over the wavelength with the Eq.(2.28) using the values from the ISO 9845 and the overall transmission is obtained as **0.195**

2.2 Single Partially Transmitting Layers with Thickness D<λ(with Wave Interference Effect)

When a window or transmitting layer is very thin, so that its thickness is comparable to the radiation wavelength, there can be interference between incident and reflected waves. For the present discussion a window where D is very close to wavelengths thick so interference effect is considered(Section 1.2.3).

The determination of the reflection and transmission coefficients for a single absorbing or non-absorbing layer, bounded on either side by semi-infinite, bounded on either side by semi-infinite layers is done by considering a beam incident on the film which is divided into reflected and transmitted parts. Such division occurs each time the beam strikes an interface so that the transmitted and reflected beams are obtained by summing the multiply-reflected and multiply-transmitted elements. For the case of single layer the summation can be computed easily. The results are conveniently expressed in terms of Fresnel's coefficients(see Eq.(1.109),(1.110),(1.111),(1.112)).[3]

A parallel beam of light of unit amplitudes and of wavelength λ_0 falls on a plane, parallel sided, homogenous, isotropic film of thickness D and refractive index n_1 supported on a substrate of index n_2. The index of first medium is n_s and angle of incidence in this medium θ.(Figure 25)

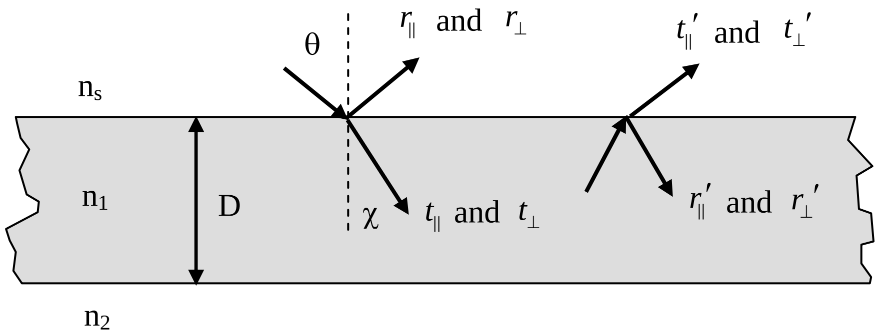


*Figure 25 **:** Layer properties and Fresnel's coefficients*

2.2.1 Non-Attenuating Single Layer

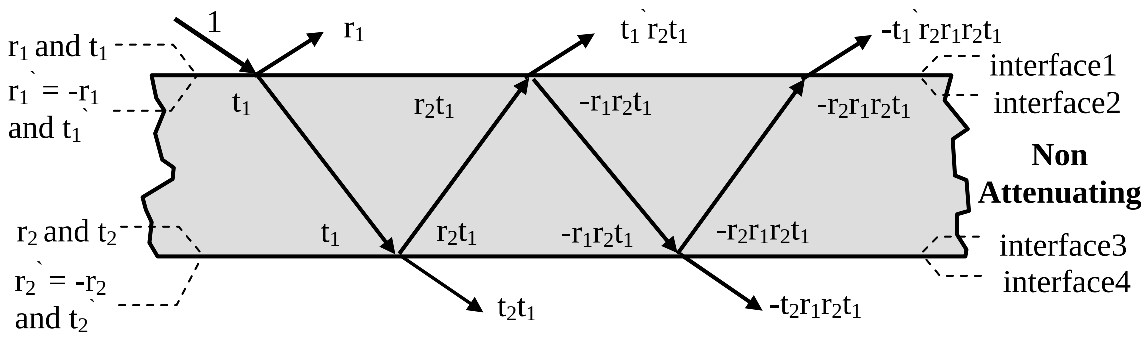


*Figure 26 **:** Multiple reflections within a thin non-attenuating film[3]*

The amplitudes of the successively reflected and transmitted beams in terms of the Fresnel's coefficients are given in the Section (1.2.3). From the definitions of these coefficients, it is clear that the values of *r* and *t* for a given boundary depend on the direction of propagation of light across the boundary.

In treating the problem of the single layer, Fresnel's coefficients for propagation from n_s to n_1 are r_1 and t_1. The corresponding coefficients for propagation from n_1 to n_s are written as r_1' and t_1'. For every coefficient, polarization has to be considered. The expressions below are valid for either polarization. Moreover, remember that according to the form of the expression for the Fresnel's reflection coefficient r_1' is equal to $-r_1$. Apart from these by taking into account the phase relationships, and defining $\gamma_1 = 4\pi n_1 D/(\lambda_0 \cos\chi)$ (see Eq.(1.104)), the reflected amplitude is the sum of the terms leaving top surface in the Figure 26,

$$R_M = r_1 + t_1 t_1' r_2 e^{-i\gamma_1} - t_1 t_1' r_1 r_2^2 e^{-2i\gamma_1} + t_1 t_1' r_1^2 r_2^3 e^{-3i\gamma_1} - .... = r_1 + \frac{t_1 t_1' r_2 e^{-i\gamma_1}}{1 + r_1 r_2 e^{-i\gamma_1}} \tag{2.34}$$

This equation can be further simplified. From conservation of energy(from Eq.(1.109)-(1.112)) it is written that,

$$t_1' t_1 = 1 - r_1^2 \tag{2.35}$$

so (2.34) can be reduced to,

$$R_M = \frac{r_1 + r_2 e^{-i\gamma_1}}{1 + r_1 r_2 e^{-i\gamma_1}} \tag{2.36}$$

The transmitted amplitude is given by,

$$T_M = t_1 t_2 e^{-i\gamma_1/2} - t_1 t_2 r_1 r_2 e^{-3i\gamma_1/2} + t_1 t_2 r_1^2 r_2^2 e^{-5i\gamma_1/2} - .... = \frac{t_1 t_2 e^{-i\gamma_1/2}}{1 + r_1 r_2 e^{-i\gamma_1}} \tag{2.37}$$

(2.36) and (2.37) takes two possible forms, depending on the state of polarization of the incident light. For light polarization with its electric vector parallel to the plane of incidence, the reflected and transmitted amplitudes are obtained by substituting for r_1, r_2, t_1, t_2 from expressions corresponding to Eq. (1.109) and (1.111). For light polarized with the electric vector perpendicular to the plane of incidence, the Fresnel coefficients as given by Eq. (1.110) and (1.112) are used.

It must be remembered that these expressions give the amplitudes of the waves in the media bounding the film. From Section (1.1.3) the wave energy depends on $|E|^2$.

Since $R_M = E_r / E_i$ (both incidence and reflected wave in the same medium which is n_s), according to the Poynting vector the reflectivity for energy is $R = |R_M|^2 = (n_s / n_s) R_M R_M^*$, where R_M^* is the complex conjugate of R_M. The reflectivity of the film is,

$$R = \frac{r_1 + r_2 e^{-i\gamma_1}}{1 + r_1 r_2 e^{-i\gamma_1}} \frac{r_1 + r_2 e^{i\gamma_1}}{1 + r_1 r_2 e^{i\gamma_1}} \tag{2.38}$$

Since $T_M = E_t / E_i$ (transmitted wave medium(n_2) and incidence wave medium(n_s) are different), according to the Poynting vector, the transmission for energy is $T = |T_M|^2 = (n_2 / n_s) T_M T_M^*$, where T_M^* is the complex conjugate of T_M. The transmission of the film is,

$$T = \frac{n_2}{n_s} \frac{t_1 t_2 e^{-i\gamma_1/2}}{1 + r_1 r_2 e^{-i\gamma_1}} \frac{t_1 t_2 e^{i\gamma_1/2}}{1 + r_1 r_2 e^{i\gamma_1}} \tag{2.39}$$

2.2.1.1 Zero reflectivity conditions

An important situation of wave interference effect is to get a low reflection from a surface. According to the relation between thickness of the layer and wavelength of the light the reflection is affected highly even it might be zero.[4]

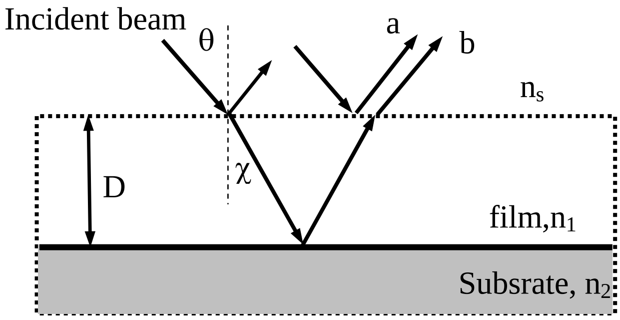


Figure 27 : Wave Interference[4]

To have zero reflected amplitude, $R_M = 0$:

Since denominator is always greater than zero, it requires that,

$$r_1 = -r_2 e^{-i\gamma_1} \tag{2.40}$$

This can be obtained if,

Condition1: $r_1 = r_2$ and $e^{-i\gamma_1} = -1$ (2.41)

OR

Condition2: $r_1 = -r_2$ and $e^{-i\gamma_1} = 1$ (2.42)

a) Normal Incidence(θ=χ=0):

For simplicity, assume that there is normal incidence wave on the film which means that incidence angle(θ = 0) is zero. And two condition will be discussed separately.

Condition 1:

To satisfy the Eq.(1.104),

$$\gamma_1 = \pi = 4\pi n_1 D / (\lambda_0 \cos(\chi)) = 4\pi n_1 D / (\lambda_0 \cos(0)) \quad (2.43)$$

From the above equations it is concluded that,

$$D = \frac{\lambda_0}{4n_1} \quad (2.44)$$

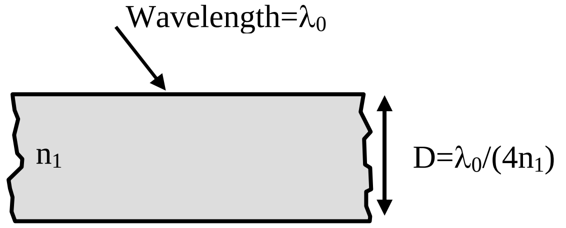


Figure 28 : Condition 1:Thickness and wavelength ratio for the zero reflectivity of normal incidence

The quantity λ_0 / n_1 is the wavelength of the radiation within the film. Hence the film thickness for zero reflection at normal incidence is one-quarter of the wavelength within the film. But also $e^{-i\gamma_1} = -1$ if $\gamma_1 = \pi, 3\pi, 5\pi, ..., (2k-1)\pi, ...$ As a result,

$$D = (2k-1)\frac{\lambda_0}{4n_1} \qquad k = 1, 2, 3, ... \quad (2.45)$$

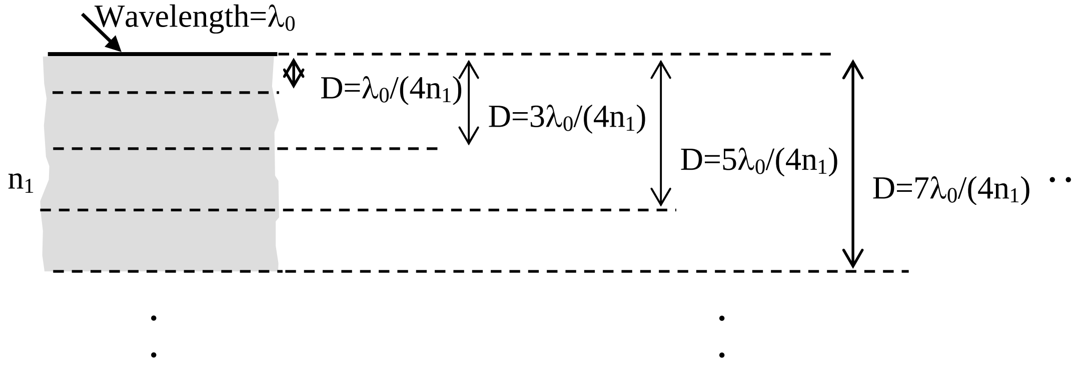


Figure 29 : Condition 1: thickness and wavelength ratio for thick layer

But this is not sufficient for zero reflection of normal incidence. There is one remaining condition. The required condition, $r_1 = r_2$, gives,

$$\frac{n_s - n_1}{n_s + n_1} = \frac{n_1 - n_2}{n_1 - n_2} \quad \text{which reduces to} \quad n_1 = \sqrt{n_s n_2}$$

Thus, for normal incidence onto a quarter-wave film from a dielectric medium with index of refraction n_s, the index of refraction of the film for zero reflection should be $n_1 = \sqrt{n_s n_2}$, the geometric mean of the *n* values on either side of the film.

Condition 2:
To satisfy the Eq.(1.104),

$$\gamma_1 = 2\pi = 4\pi n_1 D / (\lambda_0 \cos(\chi)) = 4\pi n_1 D / (\lambda_0 \cos(0)) \tag{2.46}$$

From the above equations it is concluded that,

$$D = \frac{\lambda_0}{2n_1} \tag{2.47}$$

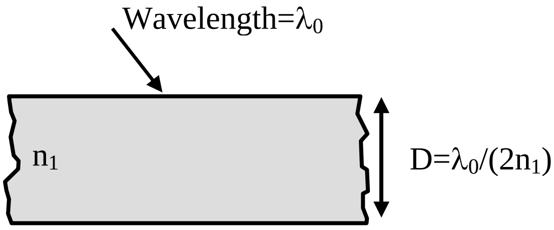


*Figure 30 **:** Condition 2:thickness and wavelength ratio for the zero reflectivity of normal incidence*

The quantity λ_0 / n_1 is the wavelength of the radiation within the film. Hence the film thickness for zero reflection at normal incidence is one-half of the wavelength within the film. Like in the Condition 1, $e^{-i\gamma_1} = -1$ if $\gamma_1 = 2\pi, 4\pi, ..., (2k)\pi, ...$ As a result,

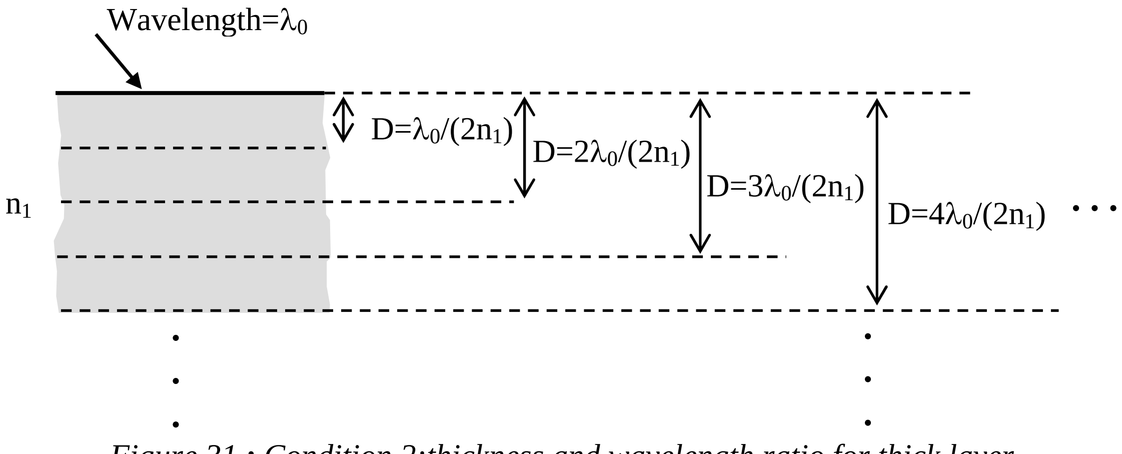


*Figure 31 **:** Condition 2:thickness and wavelength ratio for thick layer*

$$D = (k)\frac{\lambda_0}{2n_1} \qquad k = 1, 2, 3, ... \tag{2.48}$$

But this is not sufficient for zero reflection of normal incidence. There is one remaining condition. The required condition, $r_1 = -r_2$, gives,

$$\frac{n_s - n_1}{n_s + n_1} = -\frac{n_1 - n_2}{n_1 + n_2} \qquad \text{if} \qquad n_s = n_2$$

Thus, if normal incidence falls onto a half-wave film, and front and back side of the layer is same than zero reflection takes place.

However, these results are only for normal incidence at one wavelength on a non-attenuating film. To obtain more than one condition of zero reflectivity, it is necessary to use multilayer films. When the situations do not exactly satisfy the all the conditions but very close than not zero reflection occurs but very small reflection values are obtained(see the examples in the Section 2.2.4).[4]

b) With Angle of Incidence(θ>0):
If the condition become more complex like incidence with angle, than it is very hard to solve such a problem. The complexity is tried to be explained in the Condition 1. Two properties of this condition are,

$$\textit{Condition1:} \qquad r_1 = r_2 \qquad \text{and} \qquad e^{-i\gamma_1} = -1 \tag{2.49}$$

To satisfy the first property which is $e^{-i\gamma_1} = -1$,

$$\gamma_1 = \pi = 4\pi n_1 D / (\lambda_0 \cos(\chi)) \tag{2.50}$$

From the above equations it is concluded that,

$$D = \frac{\lambda_0 \cos(\chi)}{4n_1} \tag{2.51}$$

But as it is known, this is not sufficient for zero reflection. There is one more remaining condition. The required condition is $r_1 = r_2$. Because of the polarization of the incident wave $r_{1,||} = r_{2,||}$ and $r_{1,\perp} = r_{2,\perp}$.Both equality have to be considered.

To satisfy the $r_{1,||} = r_{2,||}$,

$$\frac{\cos\theta / \cos\chi - n_s / n_1}{\cos\theta / \cos\chi + n_s / n_1} = \frac{\cos\chi / \cos\alpha - n_1 / n_2}{\cos\chi / \cos\alpha + n_1 / n_2} \tag{2.52}$$

and it can be very easily seen that n_1 depends also on χ,

$$n_1 = f(\chi, \theta, n_s, n_2) \tag{2.53}$$

As a result to obtain refractive index for zero reflectivity at a constant angle of incidence(θ) angle of refraction is also needed(χ). But the problem come into picture while calculating the angle of refraction. Because during this process n_1 is also needed. So an iterative methods can be used at this stage. Apart from this, perpendicular component also have to taken into account and another equation is obtained for refractive index of the layer. And common solution of the parallel and perpendicular components are needed.

And this is out of our scope at that point so it is not considered any more. Because not the design process but the results are investigated. So this subject is also discussed in the Example 8 with commented on the results.

As the condition becomes more complex it is very hard to obtain the properties of the layers for the zero reflections. The complexity can be given also with attenuating materials and multilayered glazing. For that reason in the later sections this subject is not investigated in detail but tried to explain with the results of the examples.

2.2.2 Attenuating Single Layer

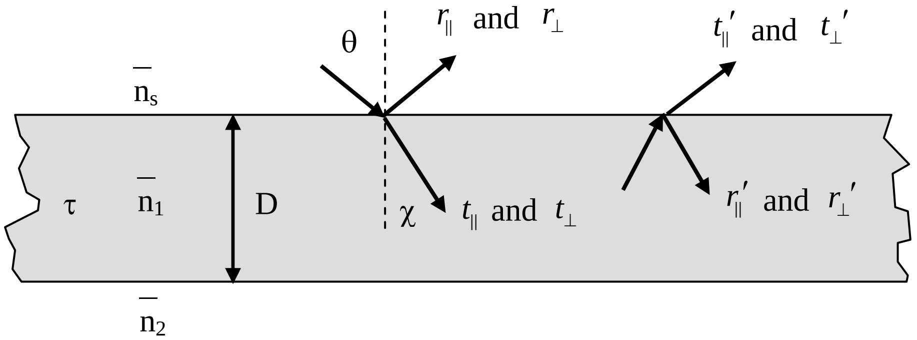


*Figure 32 **:** Layer properties and Fresnel's coefficients of attenuating medium*

The only difference according to the Section 2.2.1 is the absorption of the layer. The amplitudes of the successively reflected and transmitted beams in terms of the Fresnel's coefficients which are complex number are given in the Section (1.2.4). For layer properties and Fresnel's coefficients see the Figure 32. Fresnel's coefficients for propagation from $\bar{n}_s$ to $\bar{n}_1$ are r_1 and t_1. The corresponding coefficients for propagation from $\bar{n}_1$ to $\bar{n}_s$ are written as r_1' and t_1'. The expressions below are valid for either polarization. And also keep in mind that for the Fresnel's reflection coefficient r_1' is equal to $-r_1$. Apart from these by taking into account the phase relationships, and defining $\gamma_1 = 4\pi n_1 D/(\lambda_0 \cos\chi)$ (see Eq.(1.104)). But this equation is proofed for the non-absorbing material. If the wave attenuates in the layer refractive index(n_1-κ_1) and $\cos\chi$ (assume a+ib) become complex number. Then the phase difference turns into form of,

$$\gamma_1 = 4\pi(n_1 - i\kappa_1)D/[\lambda_0(a+bi)] \tag{2.54}$$

For the attenuating material phase difference is also complex number. And this is a problem that is very hard to solve. But if the glazing material properties are taken into consideration it is seen that extinction coefficient(κ) and imaginary term of refractive angle(b) are very close to zero. So they can be neglected without any doubt. Finally phase difference is obtained for absorbing glazing materials as,

$$\gamma_1 = 4\pi(\mathrm{Re}(\bar{n}_1))D/[\lambda_0 \,\mathrm{Re}(\cos\chi)] = 4\pi n_1 D/[\lambda_0 a] \tag{2.55}$$

The reflected amplitude is the sum of the terms leaving top surface in the Figure 33,

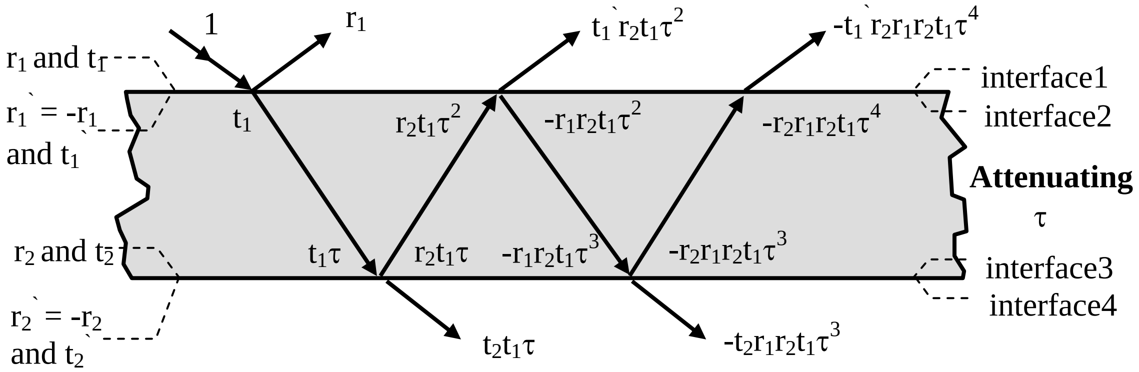


Figure 33 **:** *Multiple reflections within a thin attenuating film*

$$R_M = r_1 + t_1 t_1' r_2 \tau^2 e^{-i\gamma_1} - t_1 t_1' r_1 r_2^2 \tau^4 e^{-2i\gamma_1} + t_1 t_1' r_1^2 r_2^3 \tau^6 e^{-3i\gamma_1} - .... = r_1 + \frac{t_1 t_1' r_2 \tau^2 e^{-i\gamma_1}}{1 + r_1 r_2 \tau^2 e^{-i\gamma_1}} \quad (2.56)$$

This equation can be further simplified. From conservation of energy (or from Eq. (1.115)-(1.118)), it is written that,

$$t_1' t_1 = 1 - r_1^2 \quad (2.57)$$

so Eq.(2.56) can be reduced to,

$$R_M = \frac{r_1 + r_2 \tau^2 e^{-i\gamma_1}}{1 + r_1 r_2 \tau^2 e^{-i\gamma_1}} \quad (2.58)$$

The transmitted amplitude is given by,

$$T_M = t_1 t_2 \tau e^{-i\gamma_1/2} - t_1 t_2 r_1 r_2 \tau^3 e^{-3i\gamma_1/2} + t_1 t_2 r_1^2 r_2^2 \tau^5 e^{-5i\gamma_1/2} - .... = \frac{t_1 t_2 \tau e^{-i\gamma_1/2}}{1 + r_1 r_2 \tau^2 e^{-i\gamma_1}} \quad (2.59)$$

For light polarization with its electric vector parallel to the plane of incidence, the reflected and transmitted amplitudes are obtained by substituting for r_1, r_2, t_1, t_2 from expressions corresponding to equations (1.115) and (1.117). For light polarized with the electric vector perpendicular to the plane of incidence, the Fresnel coefficients as given by equations (1.116) and (1.118) are used.

It must be remembered that these expressions give the amplitudes of the waves in the media bounding the film. From Section (1.1.3) the wave energy depends on $|\mathrm{E}|^2$.

Since $R_M = E_r / E_i$ (both incidence and reflected wave in the same medium which is $\bar{n}_s$), according to the Poynting vector(Eq.(1.46)) the reflectivity for energy is $R = |R_M|^2 = (\bar{n}_s / \bar{n}_s) R_M R_M^*$, where R_M^* is the complex conjugate of R_M. The reflectivity of attenuating film is,

$$R = \frac{r_1 + r_2 \tau^2 e^{-i\gamma_1}}{1 + r_1 r_2 \tau^2 e^{-i\gamma_1}} \frac{r_1 + r_2 \tau^2 e^{i\gamma_1}}{1 + r_1 r_2 \tau^2 e^{i\gamma_1}} \quad (2.60)$$

Since $T_M = E_t / E_i$ (transmitted wave medium($\overline{n}_2$) and incidence wave medium($\overline{n}_s$) are different), according to the Poynting vector, the transmission for energy is $T = |T_M|^2 = (n_2 / n_s) T_M T_M{}^*$, where $T_M{}^*$ is the complex conjugate of T_M. The transmission of attenuating film is,

$$T = \frac{\overline{n}_2}{\overline{n}_s} \frac{t_1 t_2 \tau e^{-i\gamma_1/2}}{1 + r_1 r_2 \tau^2 e^{-i\gamma_1}} \frac{t_1 t_2 \tau e^{i\gamma_1/2}}{1 + r_1 r_2 \tau^2 e^{i\gamma_1}} \tag{2.61}$$

The refractive index of the front and back environment are complex if absorbing material is considered. Again for glazing materials the extinction coefficient is too small and it can be neglected. As a result the transmission of the absorbing film is,

$$T = \frac{n_2}{n_s} \frac{t_1 t_2 \tau e^{-i\gamma_1/2}}{1 + r_1 r_2 \tau^2 e^{-i\gamma_1}} \frac{t_1 t_2 \tau e^{i\gamma_1/2}}{1 + r_1 r_2 \tau^2 e^{i\gamma_1}} \tag{2.62}$$

Finally the absorption of the film is,

$$A = 1 - T - R \tag{2.63}$$

2.2.3 Averaged Properties

The solar radiation properties for the system discussed above are all implicit functions of wavelength, incident angle and polarization. To obtain the averaged values the same methods which are explained in the Section 2.1.3 are used also here. For that reason they are not explained in detail but only the required formulas are repeated.

2.2.3.1 Spectral average of the system reflection, transmission and absorption

The optical indices are not dependent on either angle of incidence or polarization, although they may be strongly dependent on wavelength. Having calculated the properties at all desired wavelengths, average properties for the glazing system can then be calculated using the general equation:

$$P = \frac{\int_a^b p(\lambda) \Phi_{solar}(\lambda) \Gamma_{solar}(\lambda) d\lambda}{\int_a^b \Phi_{solar}(\lambda) \Gamma_{solar}(\lambda) d\lambda} \tag{2.64}$$

2.2.3.2 Polarization average of the system reflection, transmission and absorption

For wave incidence on a material, polarization effects must be considered. Natural sunlight is “unpolarized” so that the angle of polarization with respect to the plane of incidence fluctuates randomly. Only the time average is measurable for any kind of property *P* which reduces to a simple average of the properties for the two polarization states:

$$P = \frac{P_{\parallel} + P_{\perp}}{2} \tag{2.65}$$

2.2.4 Some Examples

Example 4:

Aim of the example: Zero reflectivity of normal incidence for non-attenuating material is illustrated(Condition 1 in Section 2.2.1.1).

Question: Compute the reflection and transmission of the non-attenuating layer below, for the given wavelength at the incidence angle of 0 degree.

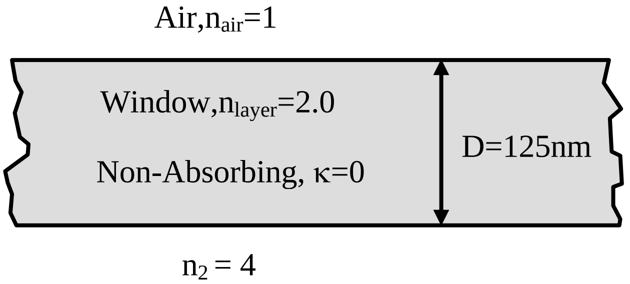


Figure 34 **:** *Properties of window for* Example 4

The Fresnel coefficients for the system are like below:

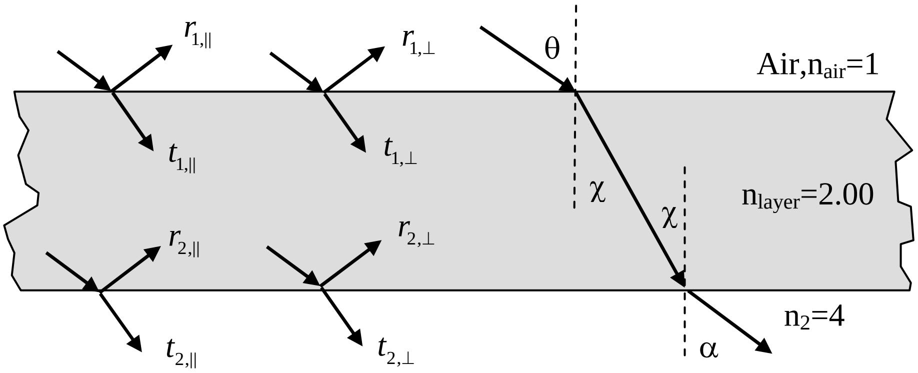


Figure 35 **:** *Fresnel's coefficients and refractive angle for* Example 4

The computation is done at wavelength λ_0 *=0.0001cm ; n=2.0 ; D=0.0000125cm :*

The angles according to the Snell's law:

$$\chi = \theta = \alpha = 0° \quad \text{(normal incidence)}$$

The phase difference is:

$$\gamma_1 = \frac{4\pi n_{layer} D \cos\chi}{\lambda_0} = \frac{4\times\pi\times 2\times 0.0000125\times\cos 0}{0.0001} = 3.14 = \pi$$

The parallel components of Fresnel's coefficients:

$$r_{1,||} = \frac{\cos\theta/\cos\chi - n_{air}/n_{layer}}{\cos\theta/\cos\chi + n_{air}/n_{layer}} = \frac{1/1-1/2}{1/1+1/2} = 0.333$$

$$t_{1,||} = \frac{2n_{air}\cos\theta}{n_{air}\cos\chi + n_{layer}\cos\theta} = \frac{2\times 1\times\cos 0}{1\times\cos 0 + 2\times\cos 0} = 0.666$$

$$r_{2,\|} = \frac{\cos\chi/\cos\alpha - n_{layer}/n_2}{\cos\chi/\cos\alpha + n_{layer}/n_2} = \frac{\cos 0/\cos 0 - 2/4}{\cos 0/\cos 0 + 2/4} = 0.333$$

$$t_{2,\|} = \frac{2n_{layer}\cos\chi}{n_{layer}\cos\alpha + n_2\cos\chi} = \frac{2\times 2\times\cos 0}{2\times\cos 0 + 4\times\cos 0} = 0.666$$

The layer parallel reflection and transmission amplitudes:

$$R_{M,\|} = \frac{r_{1,\|} + r_{2,\|}e^{-i\gamma_1}}{1 + r_{1,\|1}r_{2,\|}e^{-i\gamma_1}} = \frac{0.333 + 0.333\times e^{-i\pi}}{1 + 0.333\times 0.333\times e^{-i\pi}} = \frac{0.333 + 0.333/e^{i\pi}}{1 + 0.333\times 0.333/e^{i\pi}}$$

$$= \frac{0.333 + 0.333/(\cos\pi + i\sin\pi))}{1 + 0.333\times 0.333/(\cos\pi + i\sin\pi))} = 0$$

$$T_{M,\|} = \frac{t_{1,\|}t_{2,\|}e^{-i\gamma_1/2}}{1 + r_{1,\|}r_{2,\|}e^{-i\gamma_1}} = \frac{0.666\times 0.666\times e^{-i\pi/2}}{1 + 0.333\times 0.333\times e^{-i\pi}}$$

$$= \frac{0.666\times 0.666\times(\cos(-\pi/2) + i\sin(-\pi/2))}{1 + 0.333\times 0.333\times(\cos(-\pi) + i\sin(-\pi))} = 0.5i$$

The layer parallel reflection and transmission:

$$R_{\|} = \left|R_{M,\|}\right|^2 = (n_{air}/n_{air})R_{M,\|}R_{M,\|}^* = 0\times 0 = 0$$

$$T_{\|} = \left|T_{M,\|}\right|^2 = (n_2/n_{air})T_{M,\|}T_{M,\|}^* = \frac{4}{1}\times[(\frac{1}{2}i)\times(-\frac{1}{2}i)] = 1$$

The perpendicular components of Fresnel's coefficients:

$$r_{1,\perp} = -\frac{\cos\chi/\cos\theta - n_{air}/n_{layer}}{\cos\chi/\cos\theta + n_{air}/n_{layer}} = -\frac{\cos 0/\cos 0 - 1/2}{\cos 0/\cos 0 + 1/2} = -0.333$$

$$t_{1,\perp} = \frac{2n_{air}\cos\theta}{n_{air}\cos\theta + n_{layer}\cos\chi} = \frac{2\times 1\times\cos 0}{1\times\cos 0 + 2\times\cos 0} = 0.666$$

$$r_{2,\perp} = -\frac{\cos\alpha/\cos\chi - n_{layer}/n_2}{\cos\alpha/\cos\chi + n_{layer}/n_2} = -\frac{\cos 0/\cos 0 - 2/4}{\cos 0/\cos 0 + 2/4} = -0.333$$

$$t_{2,\perp} = \frac{2n_{layer}\cos\chi}{n_{layer}\cos\chi + n_2\cos\alpha} = \frac{2\times 2\times\cos 0}{2\times\cos 0 + 4\times 0} = 0.666$$

The layer perpendicular reflection and transmission amplitudes:

$$R_{M,\perp} = \frac{r_{1,\perp} + r_{2,\perp}e^{-i\gamma_1}}{1 + r_{1,\perp}r_{2,\perp}e^{-i\gamma_1}} = \frac{-0.333 - 0.333\times e^{-i\pi}}{1 + 0.333\times 0.333\times e^{-i\pi}} = \frac{-0.333 - 0.333/e^{i\pi}}{1 + 0.333\times 0.333/e^{i\pi}}$$

$$= \frac{-0.333 - 0.333/(\cos\pi + i\sin\pi))}{1 + 0.333\times 0.333/(\cos\pi + i\sin\pi))} = 0$$

$$T_{M,\perp} = \frac{t_{1,\perp}t_{2,\perp}e^{-i\gamma_1/2}}{1 + r_{1,\perp}r_{2,\perp}e^{-i\gamma_1}} = \frac{0.666\times 0.666\times e^{-i\pi/2}}{1 + 0.333\times 0.333\times e^{-i\pi}}$$

$$= \frac{0.666\times 0.666\times(\cos(-\pi/2) + i\sin(-\pi/2))}{1 + 0.333\times 0.333\times(\cos(-\pi) + i\sin(-\pi))} = 0.5i$$

The layer perpendicular reflection and transmission:

$$R_{\perp} = \left|R_{M,\perp}\right|^2 = (n_{air} / n_{air}) R_{M,\perp} R_{M,\perp}{}^* = 0 \times 0 = 0$$

$$T_{\perp} = \left|T_{M,\perp}\right|^2 = (n_2 / n_{air}) T_{M,\perp} T_{M,\perp}{}^* = \frac{4}{1} \times [(\frac{1}{2} i) \times (-\frac{1}{2} i)] = 1$$

For unpolarized incident radiation, one-half the energy is in each component. Hence,

$$R = \frac{R_{\parallel} + R_{\perp}}{2} = \frac{0+0}{2} = 0$$

$$T = \frac{T_{\parallel} + T_{\perp}}{2} = \frac{1+1}{2} = 1$$

Below two properties of the Condition 1 are checked(Section 2.2.1.1):

$$D = \frac{\lambda_0}{4n_1} = \frac{0.0001}{4 \times 2} = 0.0000125$$

$$n_{layer} = \sqrt{n_{air} n_2} = \sqrt{1 \times 4} = 2$$

Both condition are satisfied.

Example 5:

Aim of the example: Example 4 is repeated for the case of thicker layer(3 times of the thickness in the previous example).

Question: Compute the reflection and transmission of the non-attenuating layer below for the given wavelength at the incidence angle of 0 degree.

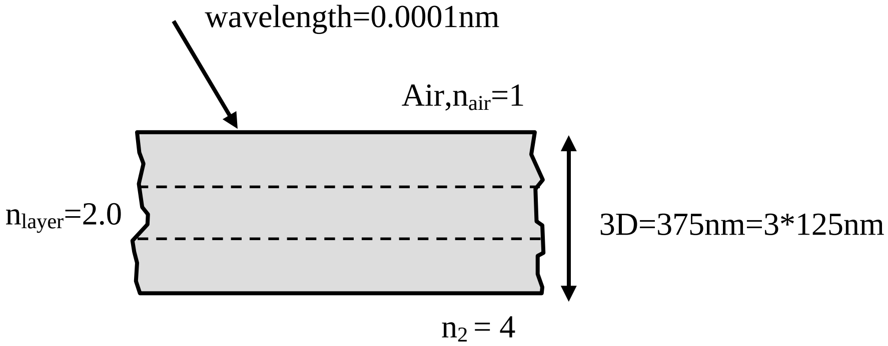


Figure 36 **:** *Properties of window for Example 5*

The procedure in the Example 4 is followed exactly. The only difference is the thickness of the layer and it becomes 3D=375nm.So except the phase difference every thing is same.

$$\chi = \theta = \alpha = 0° \quad \text{(normal incidence)}$$

The phase difference is:

$$\gamma_1 = \frac{4\pi n_{layer} 3D \cos\chi}{\lambda_0} = \frac{4 \times \pi \times 2 \times 0.0000375 \times \cos 0}{0.0001} = 3\pi$$

The parallel components of Fresnel's coefficients:

$$r_{1,\|} = \frac{\cos\theta / \cos\chi - n_{air} / n_{layer}}{\cos\theta / \cos\chi + n_{air} / n_{layer}} = \frac{1/1-1/2}{1/1+1/2} = 0.333$$

$$t_{1,\|} = \frac{2n_{air}\cos\theta}{n_{air}\cos\chi + n_{layer}\cos\theta} = \frac{2\times1\times\cos 0}{1\times\cos 0 + 2\times\cos 0} = 0.666$$

$$r_{2,\|} = \frac{\cos\chi / \cos\alpha - n_{layer} / n_2}{\cos\chi / \cos\alpha + n_{layer} / n_2} = \frac{\cos 0 / \cos 0 - 2/4}{\cos 0 / \cos 0 + 2/4} = 0.333$$

$$t_{2,\|} = \frac{2n_{layer}\cos\chi}{n_{layer}\cos\alpha + n_2\cos\chi} = \frac{2\times2\times\cos 0}{2\times\cos 0 + 4\times\cos 0} = 0.666$$

The layer parallel reflection and transmission amplitudes:

$$R_{M,\|} = \frac{r_{1,\|} + r_{2,\|}e^{-i\gamma_1}}{1 + r_{1,\|1}r_{2,\|}e^{-i\gamma_1}} = \frac{0.333 + 0.333\times e^{-i3\pi}}{1 + 0.333\times0.333\times e^{-i3\pi}} = \frac{0.333 + 0.333 / e^{i3\pi}}{1 + 0.333\times0.333 / e^{i3\pi}}$$

$$= \frac{0.333 + 0.333 / (\cos 3\pi + i\sin 3\pi))}{1 + 0.333\times0.333 / (\cos 3\pi + i\sin 3\pi))} = 0$$

$$T_{M,\|} = \frac{t_{1,\|}t_{2,\|}e^{-i\gamma_1/2}}{1 + r_{1,\|}r_{2,\|}e^{-i\gamma_1}} = \frac{0.666\times0.666\times e^{-i3\pi/2}}{1 + 0.333\times0.333\times e^{-i3\pi}}$$

$$= \frac{0.666\times0.666\times(\cos(-3\pi/2) + i\sin(-3\pi/2))}{1 + 0.333\times0.333\times(\cos(-3\pi) + i\sin(-3\pi))} = 0.5i$$

The layer parallel reflection and transmission:

$$R_{\|} = \left|R_{M,\|}\right|^2 = (n_{air} / n_{air})R_{M,\|}R_{M,\|}^{\ *} = 0\times0 = 0$$

$$T_{\|} = \left|T_{M,\|}\right|^2 = (n_2 / n_{air})T_{M,\|}T_{M,\|}^{\ *} = \frac{4}{1}\times[(\frac{1}{2}i)\times(-\frac{1}{2}i)] = 1$$

The perpendicular components of Fresnel's coefficients:

$$r_{1,\perp} = -\frac{\cos\chi / \cos\theta - n_{air} / n_{layer}}{\cos\chi / \cos\theta + n_{air} / n_{layer}} = -\frac{\cos 0 / \cos 0 - 1/2}{\cos 0 / \cos 0 + 1/2} = -0.333$$

$$t_{1,\perp} = \frac{2n_{air}\cos\theta}{n_{air}\cos\theta + n_{layer}\cos\chi} = \frac{2\times1\times\cos 0}{1\times\cos 0 + 2\times\cos 0} = 0.666$$

$$r_{2,\perp} = -\frac{\cos\alpha / \cos\chi - n_{layer} / n_2}{\cos\alpha / \cos\chi + n_{layer} / n_2} = -\frac{\cos 0 / \cos 0 - 2/4}{\cos 0 / \cos 0 + 2/4} = -0.333$$

$$t_{2,\perp} = \frac{2n_{layer}\cos\chi}{n_{layer}\cos\chi + n_2\cos\alpha} = \frac{2\times2\times\cos 0}{2\times\cos 0 + 4\times 0} = 0.666$$

The layer perpendicular reflection and transmission amplitudes:

$$R_{M,\perp} = \frac{r_{1,\perp} + r_{2,\perp}e^{-i\gamma_1}}{1 + r_{1,\perp}r_{2,\perp}e^{-i\gamma_1}} = \frac{-0.333 - 0.333\times e^{-i3\pi}}{1 + 0.333\times0.333\times e^{-i3\pi}} = \frac{-0.333 - 0.333 / e^{i3\pi}}{1 + 0.333\times0.333 / e^{i3\pi}}$$

$$= \frac{-0.333 - 0.333 / (\cos 3\pi + i\sin 3\pi))}{1 + 0.333\times0.333 / (\cos 3\pi + i\sin 3\pi))} = 0$$

$$T_{M,\perp} = \frac{t_{1,\perp} t_{2,\perp} e^{-i\gamma_1/2}}{1 + r_{1,\perp} r_{2,\perp} e^{-i\gamma_1}} = \frac{0.666 \times 0.666 \times e^{-i3\pi/2}}{1 + 0.333 \times 0.333 \times e^{-i3\pi}}$$

$$= \frac{0.666 \times 0.666 \times (\cos(-3\pi/2) + i\sin(-3\pi/2))}{1 + 0.333 \times 0.333 \times (\cos(-3\pi) + i\sin(-3\pi))} = 0.5i$$

The layer perpendicular reflection and transmission:

$$R_\perp = \left|R_{M,\perp}\right|^2 = (n_{air} / n_{air}) R_{M,\perp} R_{M,\perp}{}^* = 0 \times 0 = 0$$

$$T_\perp = \left|T_{M,\perp}\right|^2 = (n_2 / n_{air}) T_{M,\perp} T_{M,\perp}{}^* = \frac{4}{1} \times [(\frac{1}{2} i) \times (-\frac{1}{2} i)] = 1$$

For unpolarized incident radiation, one-half the energy is in each component. Hence,

$$R = \frac{R_{\|} + R_\perp}{2} = \frac{0+0}{2} = 0$$

$$T = \frac{T_{\|} + T_\perp}{2} = \frac{1+1}{2} = 1$$

Below two properties of the Condition 1 are checked(Section 2.2.1.1):

$$D = (2k-1)\frac{\lambda_0}{4n_1} = (3)\frac{0.0001}{4 \times 2} = 0.0000375cm$$

$$n_{layer} = \sqrt{n_{air} n_2} = \sqrt{1 \times 4} = 2$$

Both condition are satisfied.

Example 6:

Aim of the example: Zero reflectivity of normal incidence for non-attenuating material is illustrated(Condition 2 in Section 2.2.1.1).

Question: Compute the reflection and transmission of the non-attenuating layer below for each wavelength at the incidence angle of 0 degree. The wavelength and corresponding refractive index values are given in the Appendix.

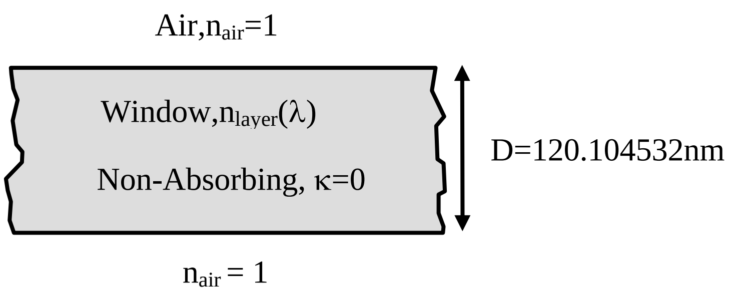


Figure 37 : Properties of window for Example 6

Only the below data set which is taken from the Appendix is discussed in detail. The solution for other wavelengths is done by the software tool. The results in detail can be found in the CD-Excel sheets-Example 6.

Wavelength(nm)	Wavelength(cm)	Refractive Index
873.22	0.000087322	3.63525

The angles according to the Snell's law:

$\chi = \theta = \alpha = 0°$ (normal incidence)

The phase difference is:

$$\gamma_1 = \frac{4\pi n_{layer} D \cos\chi}{\lambda_0} = \frac{4 \times \pi \times 3.63525 \times 0.0000120104532 \times \cos 0}{0.000087322} = 2\pi$$

The parallel components of Fresnel's coefficients:

$$r_{1,\|} = \frac{\cos\theta / \cos\chi - n_{air} / n_{layer}}{\cos\theta / \cos\chi + n_{air} / n_{layer}} = \frac{1/1 - 1/3.63525}{1/1 + 1/3.63525} = 0.569$$

$$t_{1,\|} = \frac{2n_{air} \cos\theta}{n_{air} \cos\chi + n_{layer} \cos\theta} = \frac{2 \times 1 \times \cos 0}{1 \times \cos 0 + 3.63525 \times \cos 0} = 0.431$$

$$r_{2,\|} = \frac{\cos\chi / \cos\alpha - n_{layer} / n_{air}}{\cos\chi / \cos\alpha + n_{layer} / n_{air}} = \frac{\cos 0 / \cos 0 - 3.63525/1}{\cos 0 / \cos 0 + 3.63525/1} = -0.569$$

$$t_{2,\|} = \frac{2n_{layer} \cos\chi}{n_{layer} \cos\alpha + n_{air} \cos\chi} = \frac{2 \times 3.63525 \times \cos 0}{3.63525 \times \cos 0 + 1 \times \cos 0} = 1.569$$

The layer parallel reflection and transmission amplitudes:

$$R_{M,\|} = \frac{r_{1,\|} + r_{2,\|} e^{-i\gamma_1}}{1 + r_{1,\|1} r_{2,\|} e^{-i\gamma_1}} = \frac{0.569 - 0.569 \times e^{-i2\pi}}{1 - 0.569 \times 0.569 \times e^{-i2\pi}} = \frac{0.569 - 0.569 / e^{i2\pi}}{1 - 0.569 \times 0.569 / e^{i2\pi}}$$

$$= \frac{0.569 - 0.569 / (\cos 2\pi + i \sin 2\pi))}{1 + 0.569 \times 0.569 / (\cos 2\pi + i \sin 2\pi))} = 0$$

$$T_{M,\|} = \frac{t_{1,\|} t_{2,\|} e^{-i\gamma_1/2}}{1 + r_{1,\|} r_{2,\|} e^{-i\gamma_1}} = \frac{0.431 \times 1.569 \times e^{-i2\pi/2}}{1 - 0.569 \times 0.569 \times e^{-i2\pi}}$$

$$= \frac{0.431 \times 1.569 \times (\cos(-\pi) + i \sin(-\pi))}{1 - 0.569 \times 0.569 \times (\cos(-2\pi) + i \sin(-2\pi))} = -1i$$

The layer parallel reflection and transmission:

$$R_{\|} = \left| R_{M,\|} \right|^2 = (n_{air} / n_{air}) R_{M,\|} R_{M,\|}^{\;*} = 0 \times 0 = 0$$

$$T_{\|} = \left| T_{M,\|} \right|^2 = (n_{air} / n_{air}) T_{M,\|} T_{M,\|}^{\;*} = \frac{1}{1} \times [(-1i) \times (1i)] = 1$$

The perpendicular components of Fresnel's coefficients:

$$r_{1,\perp} = -\frac{\cos\chi / \cos\theta - n_{air} / n_{layer}}{\cos\chi / \cos\theta + n_{air} / n_{layer}} = -\frac{\cos 0 / \cos 0 - 1/3.63525}{\cos 0 / \cos 0 + 1/3.63525} = -0.569$$

$$t_{1,\perp} = \frac{2n_{air} \cos\theta}{n_{air} \cos\theta + n_{layer} \cos\chi} = \frac{2 \times 1 \times \cos 0}{1 \times \cos 0 + 2 \times \cos 0} = 0.431$$

$$r_{2,\perp} = -\frac{\cos\alpha / \cos\chi - n_{layer} / n_{air}}{\cos\alpha / \cos\chi + n_{layer} / n_{air}} = -\frac{\cos 0 / \cos 0 - 3.63525/1}{\cos 0 / \cos 0 + 3.63525/1} = 0.569$$

$$t_{2,\perp} = \frac{2n_{layer}\cos\chi}{n_{layer}\cos\chi + n_{air}\cos\alpha} = \frac{2\times 3.63525\times\cos 0}{3.63525\times\cos 0 + 1\times 0} = 1.569$$

The layer perpendicular reflection and transmission amplitudes:

$$R_{M,\perp} = \frac{r_{1,\perp} + r_{2,\perp}e^{-i\gamma_1}}{1 + r_{1,\perp}r_{2,\perp}e^{-i\gamma_1}} = \frac{-0.569 + 0.569\times e^{-i2\pi}}{1 - 0.569\times 0.569\times e^{-i2\pi}} = \frac{-0.569 + 0.569 / e^{i2\pi}}{1 - 0.569\times 0.569 / e^{i2\pi}}$$

$$= \frac{-0.569 + 0.569 / (\cos 2\pi + i\sin 2\pi))}{1 - 0.569\times 0.569 / (\cos 2\pi + i\sin 2\pi))} = 0$$

$$T_{M,\perp} = \frac{t_{1,\perp}t_{2,\perp}e^{-i\gamma_1/2}}{1 + r_{1,\perp}r_{2,\perp}e^{-i\gamma_1}} = \frac{0.666\times 0.666\times e^{-i2\pi/2}}{1 - 0.569\times 0.569\times e^{-i2\pi}}$$

$$= \frac{0.431\times 1.569\times(\cos(-\pi) + i\sin(-\pi))}{1 - 0.569\times 0.569\times(\cos(-2\pi) + i\sin(-2\pi))} = -1i$$

The layer perpendicular reflection and transmission:

$$R_{\perp} = \left|R_{M,\perp}\right|^2 = (n_{air} / n_{air})R_{M,\perp}R_{M,\perp}{}^* = 0\times 0 = 0$$

$$T_{\perp} = \left|T_{M,\perp}\right|^2 = (n_2 / n_{air})T_{M,\perp}T_{M,\perp}{}^* = \frac{1}{1}\times[(-1i)\times(1i)] = 1$$

For unpolarized incident radiation, one-half the energy is in each component. Hence,

$$R = \frac{R_{\parallel} + R_{\perp}}{2} = \frac{0+0}{2} = 0$$

$$T = \frac{T_{\parallel} + T_{\perp}}{2} = \frac{1+1}{2} = 1$$

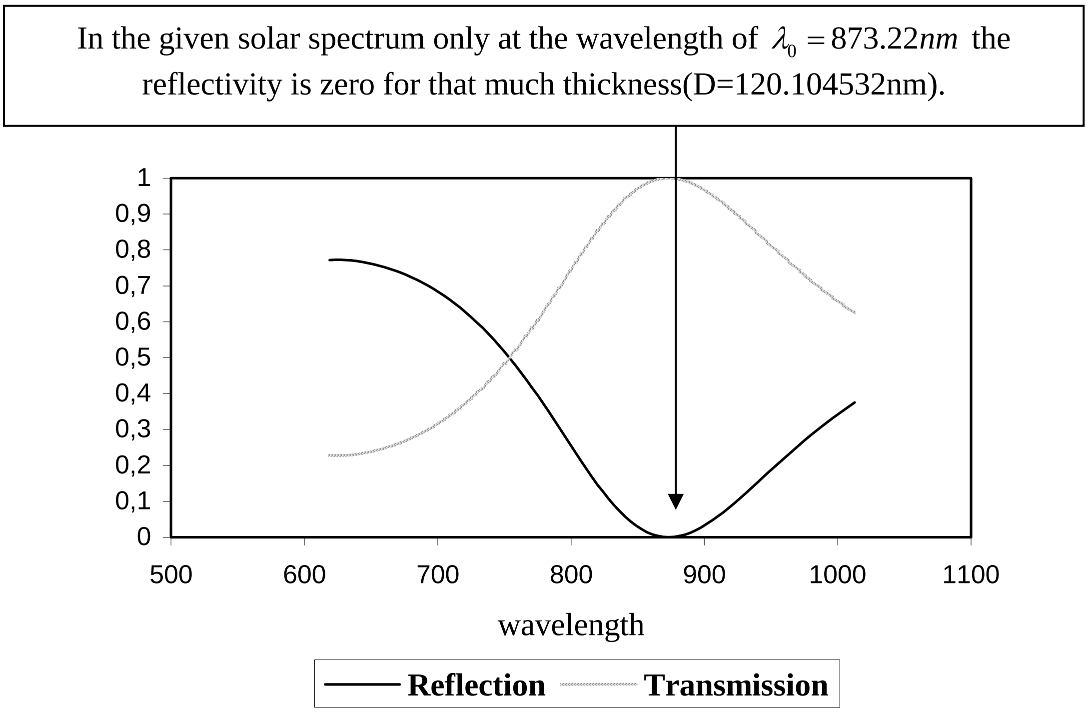
Below two properties of the Condition 2 are checked(Section 2.2.1.1):

$$D = \frac{\lambda_0}{2n_1} = \frac{0.000087322}{2\times 3.63525} = 0.0000120104532cm$$

$n_{air} = n_2 = n_{air}$ (front and back environment are same)

Both condition are satisfied.

Graph 5 is the solution for all the wavelengths. It is obtained from the software tool(see the CD-Excel sheets-Example 6).



*Graph 5 **:** Zero reflectivity for non-attenuating layer in* Example 6

Example 7:

Aim of the example: Example 6 is repeated for the layer which has a thickness of 3 times of thickness that is in the previous example.

Question: Compute the reflection and transmission of the non-attenuating layer below for each wavelength at the incidence angle of 0 degree. The thickness of the layer is 332.1472194nm. The wavelength and corresponding refractive index values which are same with the previous example are given in the Appendix.

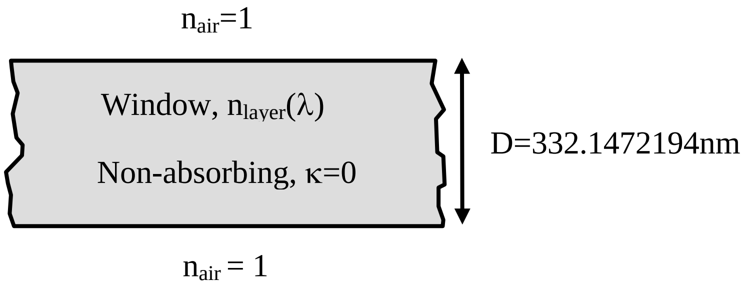
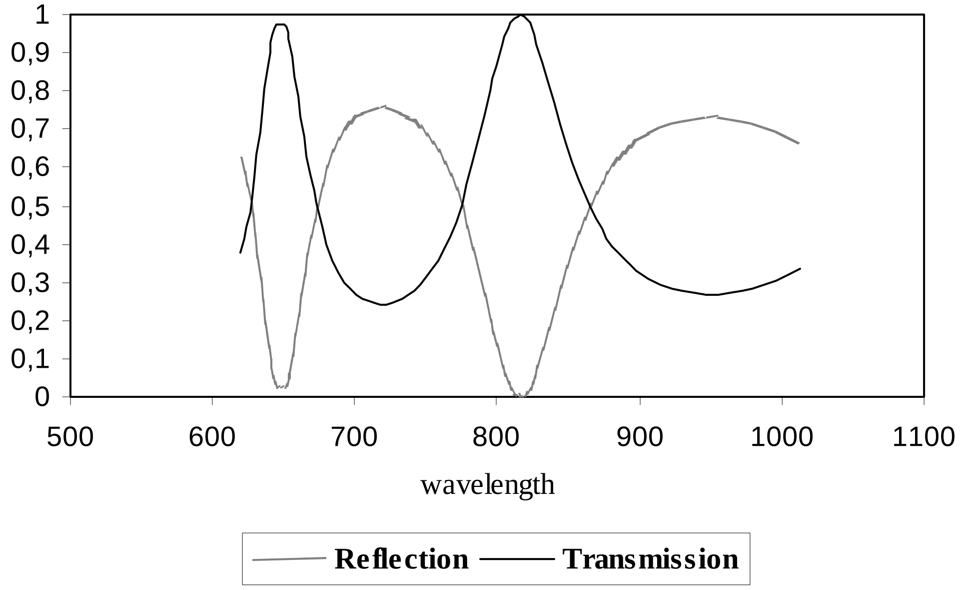


*Figure 38 **:** Properties of window for Example 7*

The procedure which is applied for the Example 6 is followed exactly for the each wavelength. So the problem is solved by software tool and all the results are in the CD-excel sheets-Example 7. Here only the graph of results are used and commented.



Graph 6 **:** *Transmission and reflection vs. wavelength for non-attenuating layer at the normal incidence in Example 7*

If it is compared with Example 6 only the thickness is changed and now at the same solar spectrum zero reflection occurs for the two points. As it can be seen form the above graph that there are two peaks. These show the wavelengths where zero or almost zero reflectivity takes place. Below data set is obtained by using software tool and gives the information at the two peak point(all the data can be found in the CD-excel sheets-Example 7),

Wavelength(nm)	Refractive Index	Transmission	Reflection
643.62 =λ_1	3.9095 =n_1	0.961193	0.0388074
651.82 =λ_2	3.895=n_2	0.970679	0.0293212
•			
•			
•			
815.82 =λ_3	3.6843=n_3	1.0	0.0

Table 4 **:** *The data set at the peak points for the Example 7*

According to the Condition 2, $D = (k)\lambda_0 / 2n_1$ or $\lambda_0 = D \times 2n_1 / k$

Consider the second peak point and assume that k=3(this is exactly the same wavelength in the data set so the zero reflectivity is obtained.),

$$\lambda_0 = D \times 2n_3 / k = 332.1472194 \times 2 \times 3.6843 / 3 = 815.82nm = \lambda_3$$

Consider the first peak point and assume that k=4,

$$\lambda_0 = D \times 2n_1 / k = 332.1472194 \times 2 \times 3.9095 / 4 = 649.26nm \approx \lambda_1 = 643.62nm$$

$$\lambda_0 = D \times 2n_2 / k = 332.1472194 \times 2 \times 3.895 / 4 = 646.87nm \approx \lambda_2 = 651.82nm$$

The wavelengths are not same with those in the data set but they are very close. For that reason at the first peak in that spectrum there is not exact wavelength that satisfies the zero reflectivity. So it is very close to zero reflectivity but not exactly.

Example 8:

Aim of the example: A complex condition is tried to be illustrated which means that explanation of a condition that is very hard to formulate for the zero reflectivity. Here the complexity comes from angle of incidence. In order to have a chance of comparison only angle of incidence of Example 7 is changed and the remaining data becomes same.

Question: Compute the reflection and transmission of the non-attenuating layer below for each wavelength at the incidence angle of 30 degree. The thickness of the layer is 332.1472194nm. The wavelength and corresponding refractive index values which are same with the previous example are given in the Appendix.

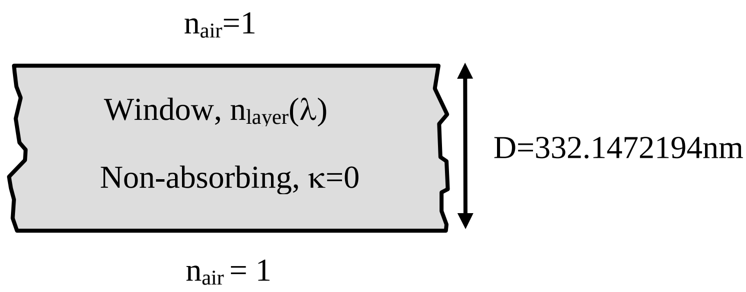
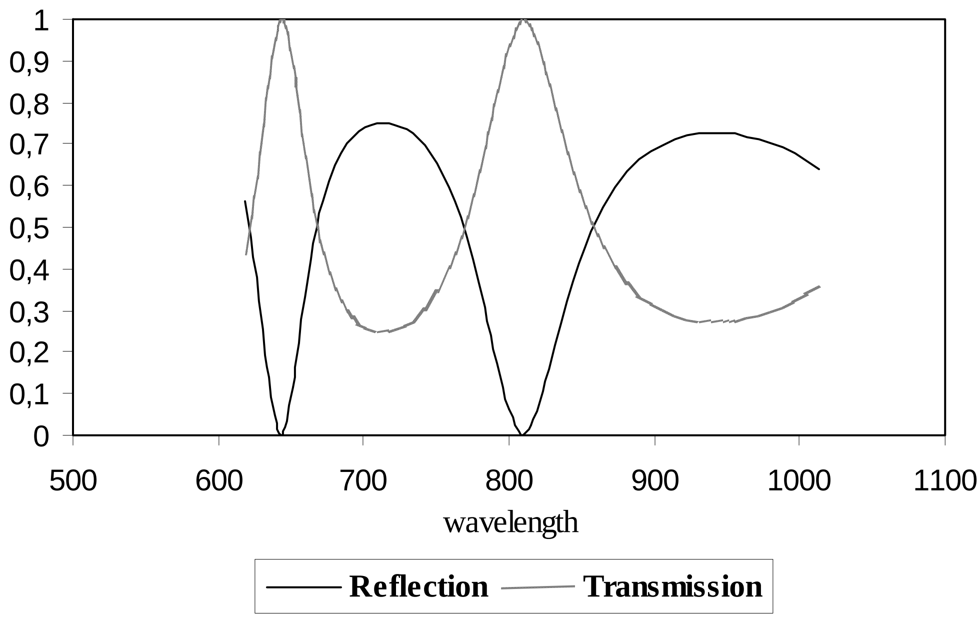


Figure 39 **:** *Properties of window for* Example 8

The reason why the angle of incidence creates complexity is at the point of giving decision to the layer's refractive index. Refractive index depends on the angle of incidence and also angle of incidence depends on the refractive index. So it is very hard to solve this problem and to decide the refractive index. The problem is solved by using software tool and the results are reached in the CD-excel sheets-Example 8. Below the transmission and reflection vs. wavelength graph is seen,



Graph 7 **:** *Transmission and reflection vs. wavelength for non-attenuating layer at the angle incidence of 30 degree in* Example 8

In the given spectrum at the two wavelength there is zero reflection. These are found at the Table 5 .

Wavelength(nm)	Refractive Index	Transmission	Reflection
643.62 =λ_1	3.9095 =n_1	1.0	0.0
.			
.			
.			
807.62 =λ_2	3.69217=n_2	1.0	0.0

Table 5 **:** *The data set at the peak points for the* Example 8

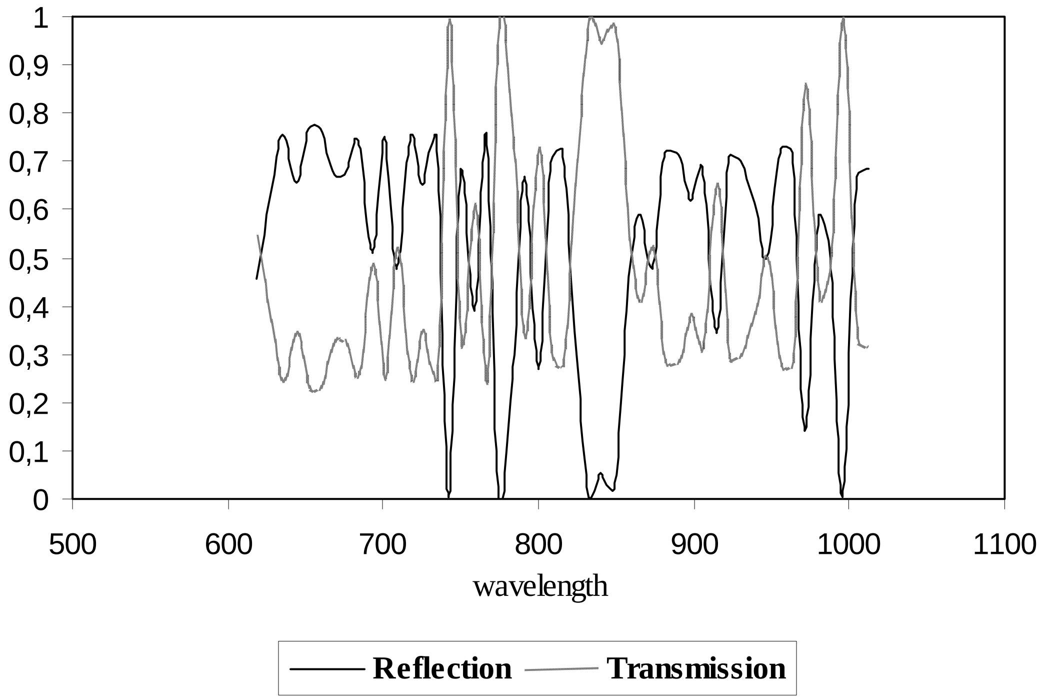
The peaks position are very close the peaks which are in the previous example but not the same(Table 4). Because of the changing only the angle of incidence, zero reflectivity occurs in a different position but not too much. This is the effect of the angle between the wave and normal of the surface.

Example 9:

Aim of the example: Example 6 is repeated for the very thick layer.

Question: Compute the reflection and transmission of the non-attenuating layer below for each wavelength at the incidence angle of 0 degree. The thickness of the layer is 1cm. The wavelength and corresponding refractive index values same with the Example 6 are given in the Appendix.

The problem is solved by using software tool and the results are reached in the CD-excel sheets-Example 9. Below the transmission and reflection vs. wavelength graph is seen,



Graph 8 **:** *Transmission and reflection vs. wavelength for non-attenuating layer at the normal incidence in* Example 9

If the thickness is very high compared to the wavelength, in general the methods that are explained in the Section 2.1 are used. But as in this example wave interference effects also can be considered. And because of the thickness, number of the wavelengths that the zero or minimum reflectivity takes place increases very much. As a result such a oscillatoric graph is obtained over the wavelength. This subject is discussed in detail in the Section 2.5

Example 10:

Aim of the example: The solution process of attenuating material is illustrated.

Question: Compute the reflection, transmission and absorption of the attenuating layer below for the given wavelength at the incidence angle of 0 degree.

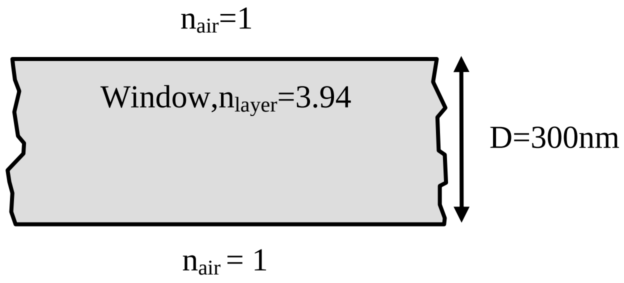


*Figure 40 **:** Properties of window for* Example 10

Wavelength(nm)	**Wavelength(cm)**	**Refractive Index**	**Extinction Coefficient**
627.22	0.000062722	3.94	7.39888E-05

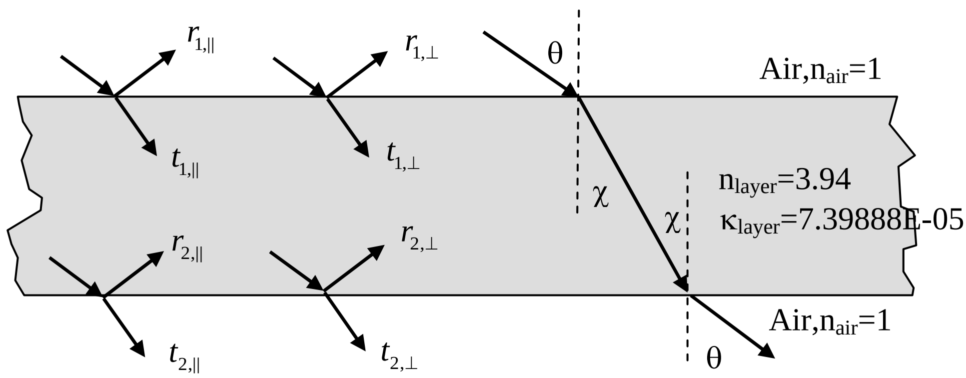


*Figure 41 **:** Fresnel's coefficients and refractive angle for* Example 10

The angles according to the Snell's law:

$$\chi = \theta = 0° \quad \text{(normal incidence)}$$

The phase difference is:

$$\gamma_1 = \frac{4\pi n_{layer} D \cos\chi}{\lambda_0} = \frac{4 \times \pi \times 3.94 \times 0.0000300 \times \cos 0}{0.000062722} = 23.68$$

Transmission in the layer under the absorption losses,

$$\tau = \exp(-4\pi\kappa \frac{x}{\lambda_0 \cos\chi}) = \exp(-4\times\pi\times 7.39888\times 10^{-5}\times \frac{0.0000300}{0.000062722\times\cos 0})$$

$$= 0.999 \approx 1$$

The thickness of the layer is very thin and the extinction coefficient of the material is too small. For these reasons the wave attenuation in the layer assumed to be almost zero.

The parallel components of Fresnel's coefficients:

$$r_{1,||} = \frac{\cos\theta / \cos\chi - (n_{air} - i\kappa_{air})/(n_{layer} - i\kappa_{layer})}{\cos\theta / \cos\chi + (n_{air} - i\kappa_{air})/(n_{layer} - i\kappa_{layer})}$$

$$= \frac{\cos 0/\cos 0 - (1-i0)/(3.94 - i7.39888\times 10^{-5})}{\cos 0/\cos 0 + (1-i0)/(3.94 - i7.39888\times 10^{-5})} = 0.595 - i6.06\times 10^{-6}$$

$$t_{1,||} = \frac{2(n_{air} - i\kappa_{air})\cos\theta}{(n_{air} - i\kappa_{air})\cos\chi + (n_{layer} - i\kappa_{layer})\cos\theta}$$

$$= \frac{2\times 1\times\cos 0}{1\times\cos 0 + (3.94 - i\,7.39888\times 10^{-5})\times\cos 0} = 0.405 + i6.06\times 10^{-6}$$

$$r_{2,||} = \frac{\cos\chi / \cos\theta - (n_{layer} - i\kappa_{layer})/(n_{air} - i\kappa_{layer})}{\cos\chi / \cos\theta + (n_{layer} - i\kappa_{layer})/(n_{air} - i\kappa_{air})}$$

$$= \frac{\cos 0/\cos 0 - (3.94 - i\,7.39888\times 10^{-5})/1}{\cos 0/\cos 0 + (3.94 - i\,7.39888\times 10^{-5})/1} = -0.595 + i6.06\times 10^{-6}$$

$$t_{2,||} = \frac{2(n_{layer} - i\kappa_{layer})\cos\chi}{(n_{layer} - i\kappa_{layer})\cos\chi + (n_{air} - i\kappa_{air})\cos\theta}$$

$$= \frac{2\times(3.94 - i\,7.39888\times 10^{-5})\times\cos 0}{(3.94 - i\,7.39888\times 10^{-5})\times\cos 0 + 1\times\cos 0} = 1.595 - i6.06\times 10^{-6}$$

The layer parallel reflection and transmission amplitudes:

$$R_{M,||} = \frac{r_{1,||} + r_{2,||}\tau^2 e^{-i\gamma_1}}{1 + r_{1,||1}r_{2,||}\tau^2 e^{-i\gamma_1}} = \frac{(0.595 - i6.06\times 10^{-6}) + 1\times(-0.595 + i6.06\times 10^{-6})\times e^{-i23.68}}{1 + 1\times(0.595 - i6.06\times 10^{-6})\times(-0.595 + i6.06\times 10^{-6})\times e^{-i23.68}}$$

$$= \frac{(0.595 - i6.06\times 10^{-6}) + (-0.595 + i6.06\times 10^{-6})\times(\cos -23.68 + i\sin 23.68))}{1 + (0.595 - i6.06\times 10^{-6})\times(-0.595 + i6.06\times 10^{-6})\times(\cos -23.68 + i\sin 23.68))}$$

$$= 0.681 - i0.365$$

$$T_{M,||} = \frac{t_{1,||}t_{2,||}\tau e^{-i\gamma_1/2}}{1 + r_{1,||}r_{2,||}\tau^2 e^{-i\gamma_1}} = \frac{1\times(0.405 + i6.06\times 10^{-6})\times(1.595 - i6.06\times 10^{-6})\times e^{-i\pi/2}}{1 + 1\times(0.595 - i6.06\times 10^{-6})\times(-0.595 + i6.06\times 10^{-6})\times e^{-i12.02}}$$

$$= \frac{(0.405 + i6.06\times 10^{-6})\times 1.595 - i6.06\times 10^{-6}\times(\cos -11.84 + i\sin 11.84)))}{1 + (0.595 - i6.06\times 10^{-6})\times(-0.595 + i6.06\times 10^{-6})\times(\cos -23.68 + i\sin 23.68)))}$$

$$= 0.301 + i0.558$$

The layer parallel reflection and transmission:

$$R_{\|} = \left| R_{M,\|} \right|^2 = (n_{air} / n_{air}) R_{M,\|} R_{M,\|}^* = (0.681 - i0.365) \times (0.681 + i0.365) = 0.6$$

$$T_{\|} = \left| T_{M,\|} \right|^2 = (n_{air} / n_{air}) T_{M,\|} T_{M,\|}^* = (0.301 + i0.558) \times (0.301 - i0.558) = 0.4$$

The perpendicular components of Fresnel's coefficients:

$$r_{1,\perp} = -\frac{\cos\chi / \cos\theta - (n_{air} - i\kappa_{air}) / (n_{layer} - i\kappa_{layer})}{\cos\chi / \cos\theta + (n_{air} - i\kappa_{air}) / (n_{layer} - i\kappa_{layer})}$$

$$= -\frac{\cos 0 / \cos 0 - (1 - i0) / (3.94 - i7.39888 \times 10^{-5})}{\cos 0 / \cos 0 + (1 - i0) / (3.94 - i7.39888 \times 10^{-5})} = -0.595 + i6.06 \times 10^{-6}$$

$$t_{1,\perp} = \frac{2(n_{air} - i\kappa_{air}) \cos\theta}{(n_{air} - i\kappa_{air}) \cos\theta + (n_{layer} - i\kappa_{layer}) \cos\chi}$$

$$= \frac{2 \times 1 \times \cos 0}{1 \times \cos 0 + (3.94 - i\,7.39888 \times 10^{-5}) \times \cos 0} = 0.405 + i6.06 \times 10^{-6}$$

$$r_{2,\perp} = -\frac{\cos\theta / \cos\chi - (n_{layer} - i\kappa_{layer}) / (n_{air} - i\kappa_{layer})}{\cos\theta / \cos\chi + (n_{layer} - i\kappa_{layer}) / (n_{air} - i\kappa_{air})}$$

$$= -\frac{\cos 0 / \cos 0 - (3.94 - i\,7.39888 \times 10^{-5}) / 1}{\cos 0 / \cos 0 + (3.94 - i\,7.39888 \times 10^{-5}) / 1} = 0.595 - i6.06 \times 10^{-6}$$

$$t_{2,\perp} = \frac{2(n_{layer} - i\kappa_{layer}) \cos\chi}{(n_{layer} - i\kappa_{layer}) \cos\chi + (n_{air} - i\kappa_{air}) \cos\theta}$$

$$= \frac{2 \times (3.94 - i\,7.39888 \times 10^{-5}) \times \cos 0}{(3.94 - i\,7.39888 \times 10^{-5}) \times \cos 0 + 1 \times \cos 0} = 1.595 - i6.06 \times 10^{-6}$$

The layer perpendicular reflection and transmission amplitudes:

$$R_{M,\perp} = \frac{r_{1,\perp} + \tau^2 r_{2,\perp} e^{-i\gamma_1}}{1 + r_{1,\perp} r_{2,\perp} \tau^2 e^{-i\gamma_1}} = \frac{(-0.595 + i6.06 \times 10^{-6}) + 1 \times (0.595 - i6.06 \times 10^{-6}) \times e^{-i23.68}}{1 + 1 \times (-0.595 + i6.06 \times 10^{-6}) \times (0.595 - i6.06 \times 10^{-6}) \times e^{-i23.68}}$$

$$= \frac{(-0.595 + i6.06 \times 10^{-6}) + (0.595 - i6.06 \times 10^{-6}) \times (\cos -23.68 + i \sin 23.68))}{1 + (-0.595 + i6.06 \times 10^{-6}) \times (0.595 - i6.06 \times 10^{-6}) \times (\cos -23.68 + i \sin 23.68))}$$

$$= -0.681 + i0.365$$

$$T_{M,\perp} = \frac{t_{1,\perp} t_{2,\perp} \tau e^{-i\gamma_1/2}}{1 + r_{1,\perp} r_{2,\perp} \tau^2 e^{-i\gamma_1}} = \frac{1 \times (0.405 + i6.06 \times 10^{-6}) \times 1.595 - i6.06 \times 10^{-6} \times e^{-i\pi/2}}{1 + 1 \times (-0.595 + i6.06 \times 10^{-6}) \times (0.595 - i6.06 \times 10^{-6}) \times e^{-i12.02}}$$

$$= \frac{(0.405 + i6.06 \times 10^{-6}) \times 1.595 - i6.06 \times 10^{-6} \times (\cos -11.84 + i \sin 11.84)))}{1 + (-0.595 + i6.06 \times 10^{-6}) \times (0.595 - i6.06 \times 10^{-6}) \times (\cos -23.68 + i \sin 23.68)))}$$

$$= 0.301 + i0.558$$

The layer perpendicular reflection and transmission:

$$R_{\perp} = \left| R_{M,\perp} \right|^2 = (n_{air} / n_{air}) R_{M,\perp} R_{M,\perp}^* = (-0.681 + i0.365) \times (-0.681 - i0.365) = 0.6$$

$$T_{\perp} = \left|T_{M,\perp}\right|^2 = (n_{air} / n_{air}) T_{M,\perp} T_{M,\perp}^{*} = (0.301 + i0.558) \times (0.301 - i0.558) = 0.4$$

For unpolarized incident radiation, one-half the energy is in each component. Hence,

$$R = \frac{R_{\parallel} + R_{\perp}}{2} = \frac{0.6 + 0.6}{2} = 0.6$$

$$T = \frac{T_{\parallel} + T_{\perp}}{2} = \frac{0.4 + 0.4}{2} = 0.4$$

$A = 1 - 0.6 - 0.4 = 0$ (because absorption losses are assumed zero)

Example 11:

Aim of the example: The Example 10 is repeated for all wavelengths in a given solar spectrum which can be found in the Appendix.

Question: Compute the reflection, transmission and absorption of the attenuating layer below for the given wavelength spectrum at the incidence angle of 0 degree.

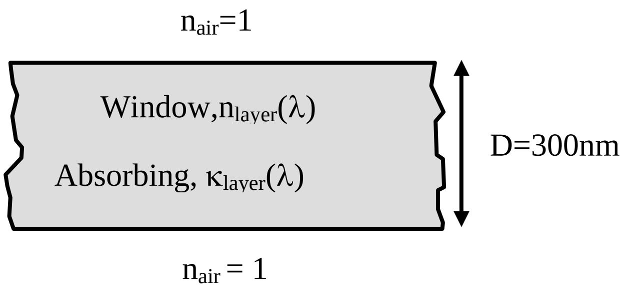
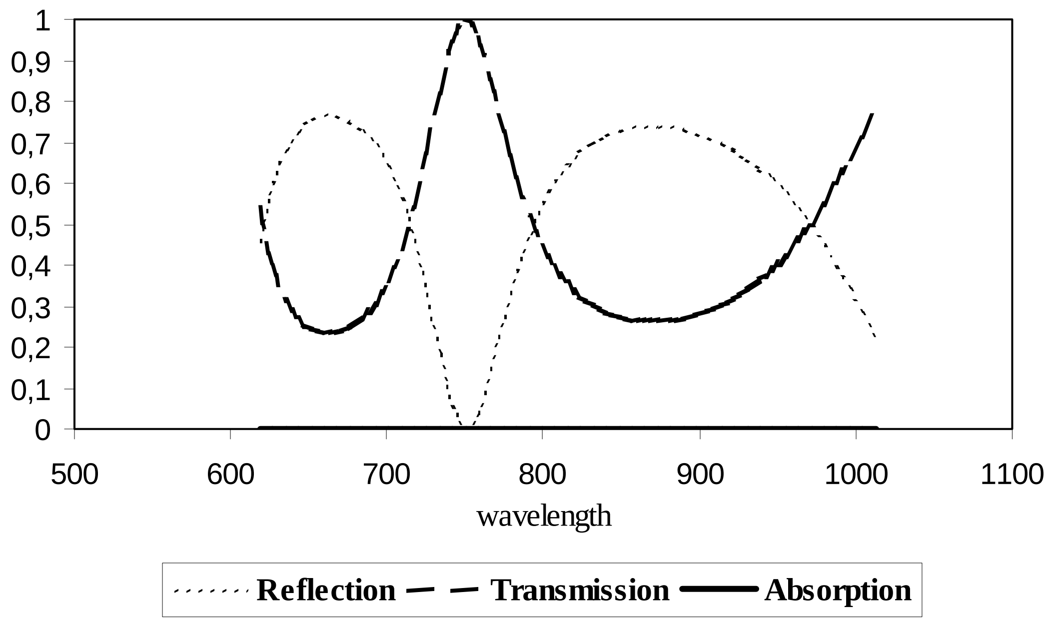


Figure 42 ***:*** *Properties of window for* Example 11

The procedure which is applied for the Example 10 is followed exactly for the each wavelength. So the problem is solved by software tool and all the results are in the CD-excel sheets-Example 11. Here only the results are used and commented.



Graph 9 ***:*** *Transmission, reflection, absorption vs. wavelength for attenuating layer at the normal incidence in Example 11*

At the two wavelength, the data sets are explained. These are the wavelength which is explained in the previous example and the zero reflection point.

Wavelength(nm)	**Refr. Index**	**Extinction Coeff.**	**Trans.**	**Ref.**	**Abs.**
627.22 $=\lambda_1$	3.94 $=n_1$	7.39888E-05 $=\kappa_1$	**0.4**	**0.6**	**0.0**

Table 6 **:** *The data set at the wavelength λ=627.22nm for the* Example 11

The same results are reached for the transmission, reflection and absorption with the previous example.

Wavelength(nm)	**Refr. Index**	**Extinction Coeff.**	**Trans.**	**Ref.**	**Abs.**
750.22 $=\lambda_1$	3.7537 $=n_1$	0.0000376375 $=\kappa_1$	**1.0**	**0.0**	**0.0**

Table 7 **:** *The data set at the wavelength λ=750.22nm which is zero reflectivity point for the* Example 11

At the wavelength 750.22nm zero reflection takes place. For the condition of this example it is very hard to solve the zero reflection phenomena because of attenuating material. Attenuation property causes complex refractive index and complex refractive angles so the properties of the zero reflectivity for the absorbing materials are not investigated in detail. Only while the explanation of the results that subject is used.

Because the thickness of the layer is very thin and also extinction coefficients which determines the attenuation of the wave is very low, the transmission under the absorption losses is assumed 1. For that reason at both wavelength the absorption is calculates as zero.

Example 12:

Aim of the example: The Example 11 is repeated for very thick layer.

Question: Compute the reflection and transmission of the attenuating layer below for each wavelength at the incidence angle of 0 degree. The thickness of the layer is 1cm. The wavelength and corresponding refractive index values given in the Appendix same with the Example 11.

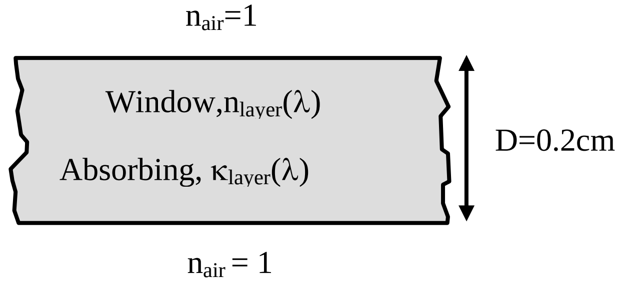
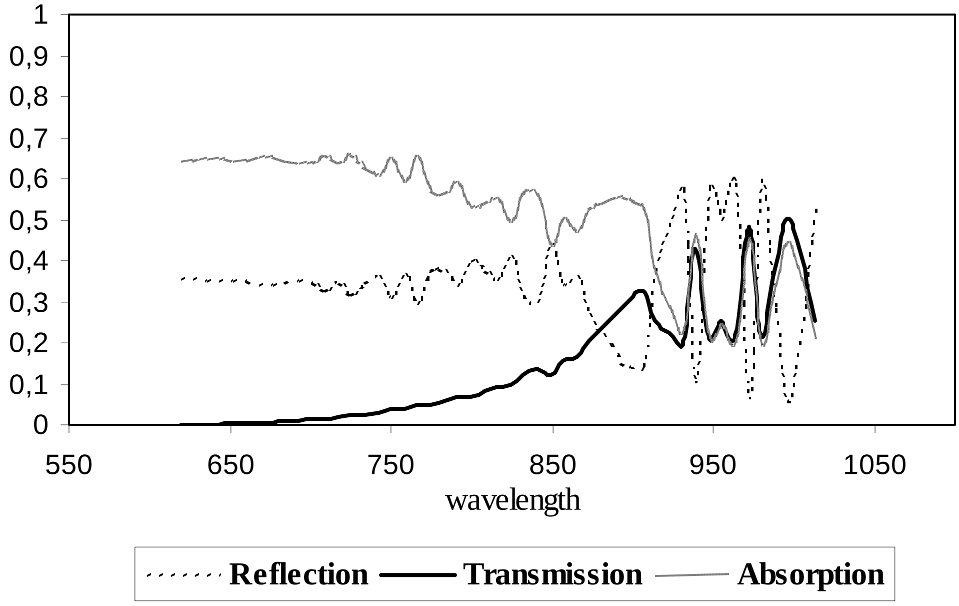


Figure 43 **:** *Properties of window for Example 12*

The problem is solved by using software tool and the results are reached in the CD-excel sheets-Example 12. In the Graph 10 the transmission, reflection, and absorption vs. wavelength is seen,



Graph 10 **:** *Transmission, reflection, absorption vs. wavelength for attenuating layer at the normal incidence in Example 12*

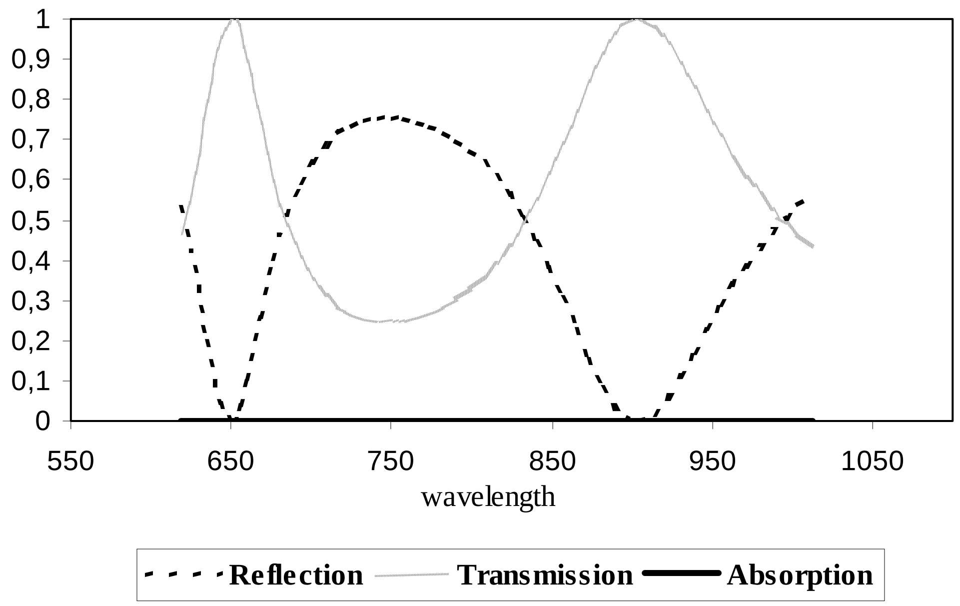
This example resembles the Example 9. Both are for the very thick layers (D>λ). If the thickness is very high compared to the wavelength then in general the methods that are explained in the Section 2.1 are used. But as in this example wave interference effects also can be considered. And because of the thickness, number of the wavelengths that the zero or minimum reflectivity takes place increases very much. As a result such a oscillatoric graph is obtained over the wavelength. The comparison of the two method is discussed in the Section 2.5.

Example 13:

Aim of the example: Up to now the conditions are considered either for one wavelength or for the wavelength spectrum at a specific angle of incidence. So spectral average is not illustrated. In this example after calculated the radiation coefficients at each wavelengths the results are averaged through the wavelength.

Question: Compute the reflection, transmission, and absorption of the attenuating layer below at the incidence angle of 0 degree. The thickness of the layer is 250nm. The wavelength and corresponding refractive index and extinction coefficient values are given in the Appendix.

The problem is solved by using software tool and the results are reached in the CD-excel sheets-Example 13. First of all, in the Graph 11 the transmission, reflection, and absorption vs. wavelength is seen,



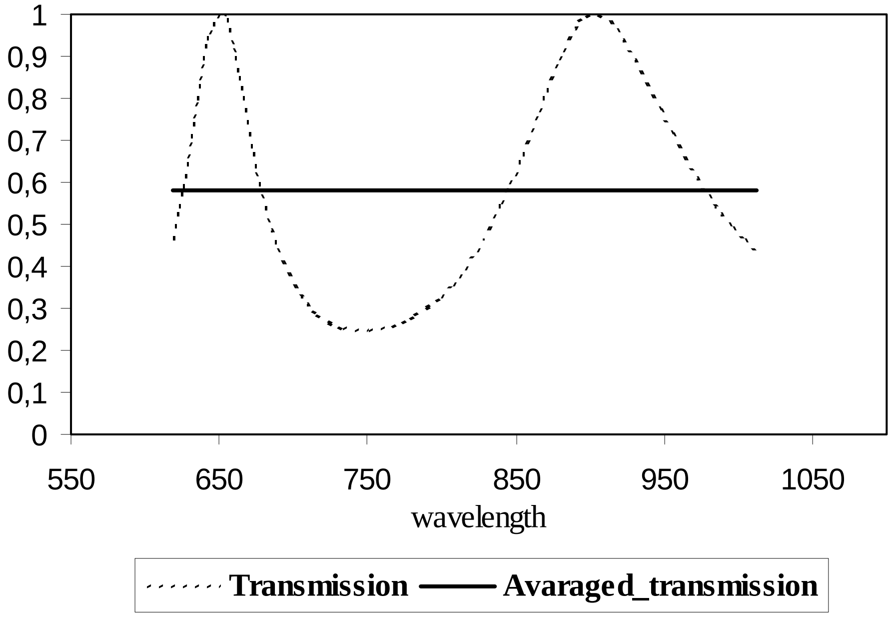
Graph 11 ***:*** *Transmission, reflection, absorption vs. wavelength for attenuating layer at the normal incidence in* Example 13

Transmission, reflection, and absorption are the calculated spectral radiometric properties. Because of the very thin layer and the small extinction coefficient absorption is zero. Remaining properties(p(λ)) which are reflection and transmission are averaged in the wavelength spectrum. The wavelengths limits are specified by 619.02nm and 1012.62nm(see Appendix). The weighting function, Φ(λ), is air mass 1.5 terrestrial solar global spectral irradiance (W/m2-micron) on a 37 tilted surface (AM 1.5 global irradiance [ISO 9845/ASM 892], see Appendix). Γ, a weighting function for the response of the "detector", is known as 1.(see Table 2)The averaged values are calculated by using the Eq.(2.28) ,

$$P = \frac{\int_a^b p(\lambda)\Phi_{solar}(\lambda)\Gamma_{solar}(\lambda)d\lambda}{\int_a^b \Phi_{solar}(\lambda)\Gamma_{solar}(\lambda)d\lambda} \tag{2.66}$$

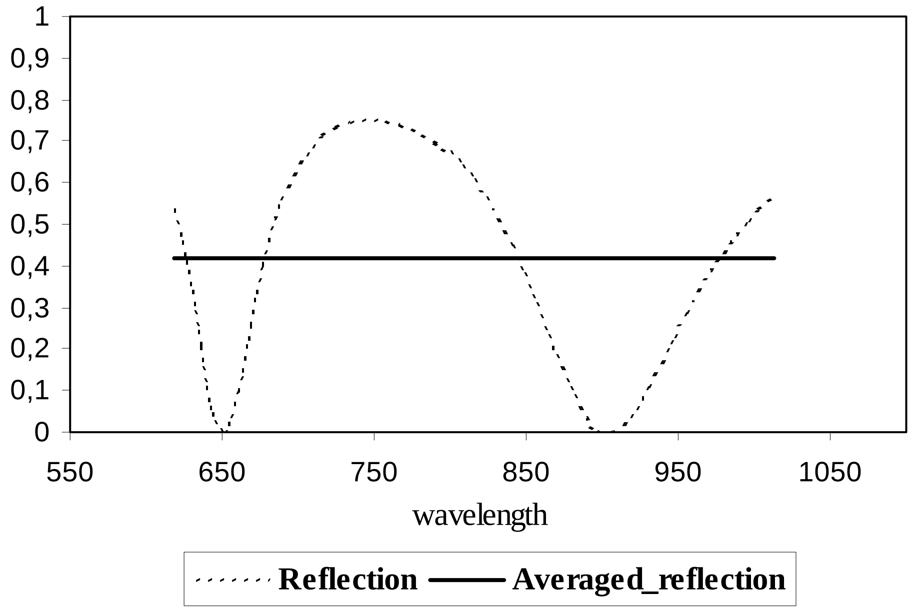
Here as explained in the Section 2.1.3.1 master function is Φ(λ), and the slave equations are transmission and reflection data set in the given wavelength spectrum. So first of all, to multiply the p(λ) and Φ(λ), by considering the master function wavelength data set the reflection and transmission properties are linearly interpolated. Then multiplication is done for every wavelength in the given wavelength range which is specified by a and b in the above equation. Finally, the integration is calculated using 2-point Newton-Cotes formula called the trapezoidal rule.

The averaged transmission value is 0.58,



*Graph 12 **:** Spectral averaged of the transmission Example 13*

The averaged reflection value is 0.42,



*Graph 13 **:** Spectral averaged of the reflection Example 13*

2.3 Partially Transmitting Multi-Layer with Thickness D>λ(No Wave Interference Effect)

In a multilayered system, the ray tracing and net radiation methods used to develop the equations in the Section 2.1 are considered again to derive the appropriate equations. Ray tracing method is generalized to any number or type of layers, and modern heat transfer calculation method has also been applied to these complicated situations.[4]

2.3.1 Ray Tracing Method

For multilayer discussion single layer transmission, reflection, and absorption properties must be remembered in this method (Section 2.1.1.1).

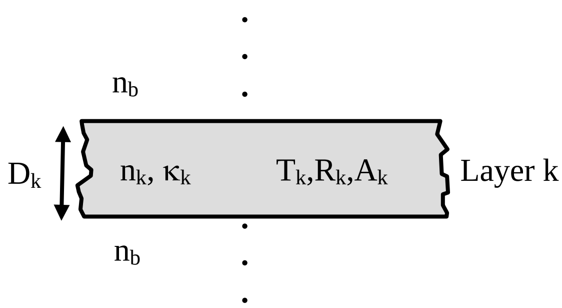


Figure 44 : Each layer's transmission, reflection and absorption properties in multilayered system

In Figure 44 it is seen that T_k, R_k, A_k are the solar radiation properties of the k^{th} layer. These are calculated for each layer in the multilayer system using the Eq.(2.1),(2.2), and (2.3) considering the front and back environment.

2.3.1.1 Non-attenuating Materials

Because of non-attenuation material property, absorption coefficient of the layer is assumed zero in that section. Firstly two layered system is discussed than the results is generalized for the multilayer system.

Two layer system:

In the Figure 45 there are so many possible reflection paths. So it might seem that ray tracing would be very complicated. However the analysis by ray tracing can be organized easily. An amount of $R_1$of the incident unit energy is reflected from the first layer, and amount T_1 is transmitted. Both T_1 and R_1 are calculated with the single layer equations considering the front and back side medium of the film(Section 2.1.1.1). In this discussion as it is seen in the figure below both medium(n_a) are the same for both layer. And T_2 and R_2 are properties of the second layer.

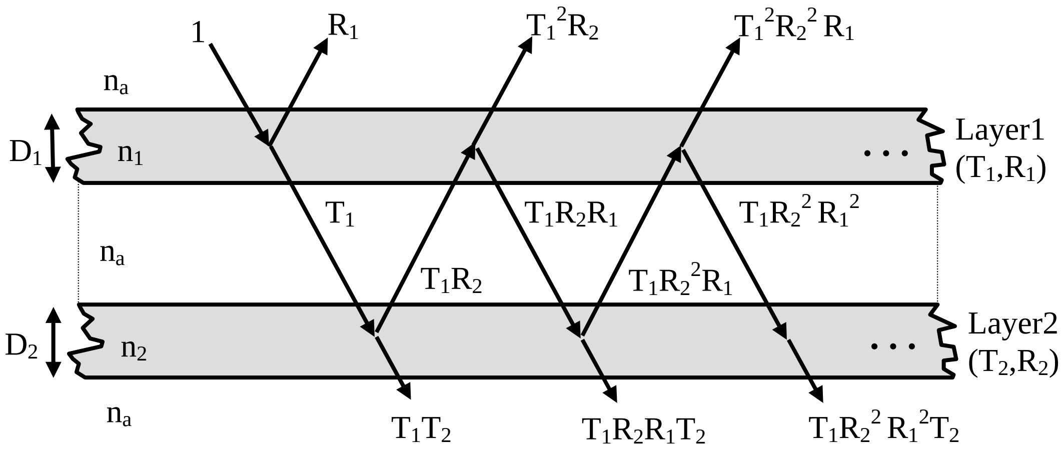


*Figure 45 **:** Ray-tracing method for parallel non-attenuating two layered system*

In Figure 45 summing the reflected and the transmitted terms give the system properties,

$$R = R_1 + R_2 T_1^2 (1 + R_1 R_2 + R_1^2 R_2^2 + ...) = R_1 + \frac{R_2 T_1^2}{1 - R_1 R_2} \quad (2.67)$$

$$T = T_1 T_2 (1 + R_1 R_2 + R_1^2 R_2^2 + ...) = \frac{T_1 T_2}{1 - R_1 R_2} \quad (2.68)$$

Now the above equations is generalized for the multilayer system.

Multi layer system:

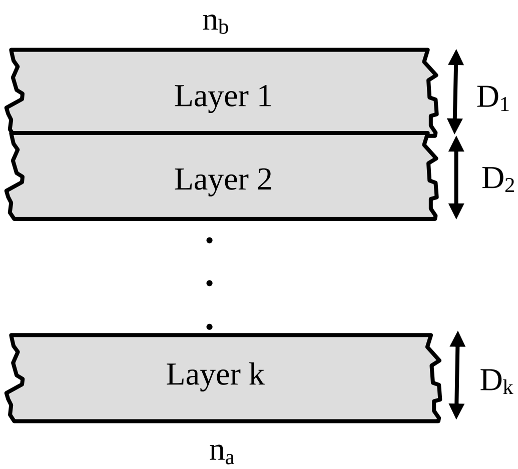


*Figure 46 **:** Ray-tracing method for multi parallel non-attenuating layer*

Equations (2.67) and (2.68) can be used to calculate the transmittance and reflectance of any number of layers by repeated application. If subscript 1 refers to the properties of a system and subscript 2 to the properties of an additional layer placed at internal side, then these equations yield the appropriate transmittance and reflectance of the new system.

At that point the most important thing is generally front and back environment of the film is different. During the calculation of the each layer properties this have to be kept in mind.

Because if this is the situation, reflection and transmission from the front side are different according to the back side like in the Figure 47(see Section 2.1.1.1). This effect have to be considered during the calculation of system properties.

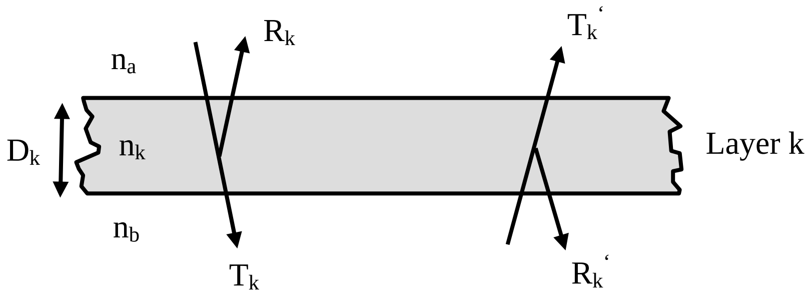


Figure 47 : Reflection and transmission of the non-attenuating single layer if front and back mediums are different

2.3.1.2 Attenuating Materials

Again also in this section firstly two layered system is discussed than the results is generalized for the multilayer system.

Two layer system:

In the Figure 48, R_1 of the incident unit energy is reflected from the first layer, T_1 is transmitted, and A_1 is absorbing part. All three are calculated with the single layer equations considering the front and back side medium of the film(Section 2.1.1.2). Also in that discussion both medium(n_a) are the same for both layer. And T_2, R_2, A_2 are properties of the second layer.

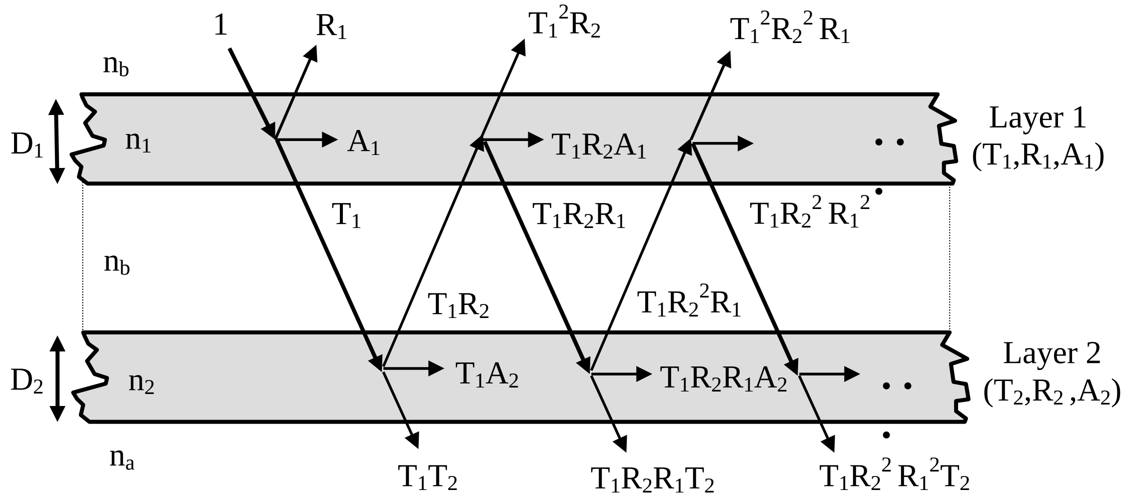


Figure 48 : Ray-tracing method for parallel attenuating two layered system

In Figure 48, summing the reflected and the transmitted terms give the system properties. The absorbed energy can be found subtracting the reflection and transmission values from the incoming wave energy which is assumed as 1.

$$R = R_1 + R_2T_1^2(1 + R_1R_2 + R_1^2R_2^2 + ...) = R_1 + \frac{R_2T_1^2}{1 - R_1R_2} \tag{2.69}$$

$$T = T_1 T_2 (1 + R_1 R_2 + R_1^2 R_2^2 + ...) = \frac{T_1 T_2}{1 - R_1 R_2} \qquad (2.70)$$

$$A = 1 - T - R \qquad (2.71)$$

Now the above equations is generalized for the multilayer system.

Multi layer system:

Just like the previous section, the Eq.(2.69), (2.70), and (2.71) can be used to calculate the transmittance and reflectance of any number of layers by repeated application. If subscript 1 refers to the properties of a system and subscript 2 to the properties of an additional layer placed at internal side, then these equations yield the appropriate transmittance and reflectance of the new system.

The problem of different front and back side environment is available also in this discussion. So explanation which is done in the previous part have to kept in mind. This is the part that the ray tracing method is complicated. But in the modern heat transfer calculation methods there are not that kind of problematic parts. For that reason the equations for this condition are proofed in the net radiation method.

2.3.2 Net radiation Method

In many situations, net radiation is easier to apply than the ray tracing method. Now the more general configuration which is not considered in the ray tracing method will be discussed with the net radiation method. This configuration consists of all glazing conditions. As it is seen in the Figure 49 there can be any number and type of the layer in the system and also front and back mediums are not the topic of special discussion. They are not important through the solution .

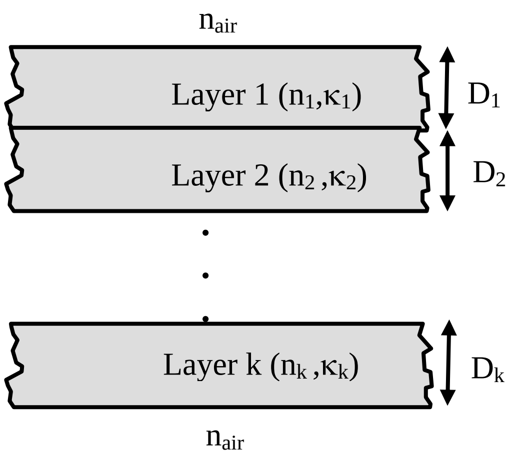


*Figure 49 **:** The general multilayer condition for net radiation method*

2.3.2.1 Non-Attenuating Materials

Before discussing the any number of layer system in order to be explain the topic in a simple condition firstly two layered system is discussed than the results is generalized for the multilayer system.

Two layer system:

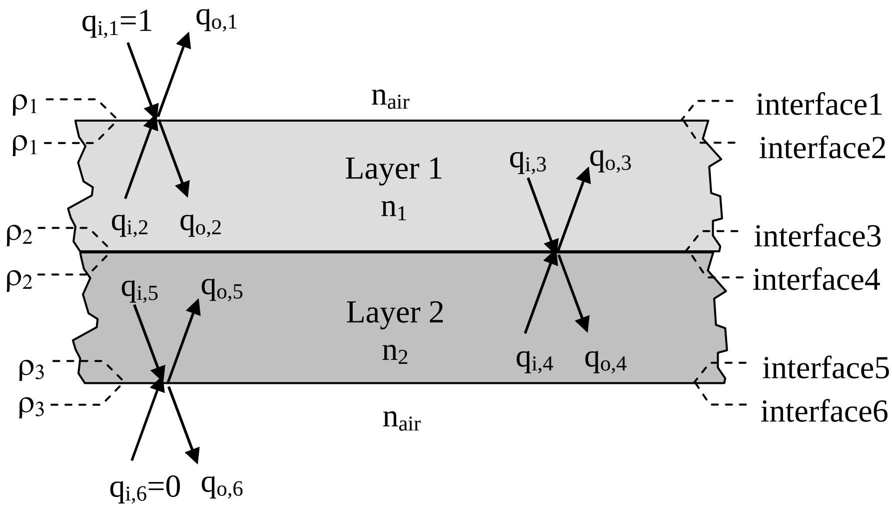


Figure 50 : Net radiation method for non-attenuating two layer system

In Figure 50, the surface reflections and notation of outgoing- ingoing fluxes are given. These are explained in detail in the Section 2.1.2.1. Energy balance equations means outgoing radiation terms in terms of the incoming fluxes for every interface are written as,

$$q_{0,1} = \rho_1 q_{i,1} + (1-\rho_1) q_{i,2} = \rho_1 + (1-\rho_1) q_{i,2} \tag{2.72}$$

$$q_{0,2} = (1-\rho_1) q_{i,1} + \rho_1 q_{i,2} = (1-\rho_1) + \rho_1 q_{i,2} \tag{2.73}$$

$$q_{0,3} = \rho_2 q_{i,3} + (1-\rho_2) q_{i,4} \tag{2.74}$$

$$q_{0,4} = (1-\rho_2) q_{i,3} + \rho_2 q_{i,4} \tag{2.75}$$

$$q_{0,5} = \rho_3 q_{i,5} + (1-\rho_3) q_{i,6} = \rho_3 q_{i,5} \tag{2.76}$$

$$q_{0,6} = (1-\rho_3) q_{i,5} + \rho_3 q_{i,6} = (1-\rho_3) q_{i,5} \tag{2.77}$$

Non-attenuating material property is have be considered to solve the above equations system. Since there is no any energy loss because of absorption all the wave intensity is transmitted from top surface to the bottom surface and vice versa. As a results,

$$q_{0,3} = q_{i,2} \quad \text{and} \quad q_{0,2} = q_{i,3} \tag{2.78}$$

$$q_{0,5} = q_{i,4} \quad \text{and} \quad q_{0,4} = q_{i,5} \tag{2.79}$$

At that point, 10 equations are obtained from the energy balance. The surface reflectance and surface transmission are known. A unit incoming flux($q_{i,1}$=1) from a single direction comes to surface 1 and a zero incoming flux($q_{i,6}$=0) comes to surface 3 which means that there is no radiation form inside to outside. And the remaining 10 unknowns are $q_{o,1}$, $q_{i,2}$, $q_{o,2}$, $q_{i,3}$, $q_{o,3}$, $q_{i,4}$, $q_{o,4}$, $q_{i,5}$, $q_{o,5}$, $q_{o,6}$. If 10 equations and 10 unknowns are solved than solar radiation properties can be obtained. Because as it can be seen in the Figure 50, $q_{o,1}$ is the system reflection and the $q_{o,6}$ is the system transmission. For the solution process Gauss elimination method is used. Explanation of the Gauss elimination method can be found in the Appendix.

If the procedure explained in the Appendix is applied to the 10 energy balance equations, reflection and transmission of the system is obtained. This step is programmed in the software tool as a separate class. Below the function that is written for Gauss elimination can be found.

```
// Gaussian elimination A x = bb

void Mtx::GaussElim(Vcr& bb) const
{
        if (dimn != bb.size() )
                error("matrix or vector sizes do not match");
        Mtx tmpx = *this;

// Forward elimination of unknowns

        for (int k = 0; k < dimn - 1; k++) {
                if (tmpx[k][k] == 0)
                error("pivot is zero in Mtx::GaussElim()");
                for (int i = k + 1; i < dimn; i++) {
                    if (tmpx[i][k] != 0) {
                        double mult = tmpx[i][k]/tmpx[k][k];
                        tmpx[i][k] = mult;
                        for (int j = k + 1; j < dimn; j++)
                                tmpx[i][j] -= mult*tmpx[k][j];
                    }
                 }
        }
        for (int i = 1; i < dimn; i++)
                for (int j = 0; j < i; j++)     bb[i] -= tmpx[i][j]*bb[j];

//Back substitution

        for (i = dimn - 1; i >= 0; i--) {
                for (int j=i+1; j<dimn; j++)    bb[i] -= tmpx[i][j]*bb[j];
                        bb[i] /= tmpx[i][i];
        }
} // end GaussElim()
```

Table 8 : Programming of the gauss elimination method in the software tool

The equations between (2.72)-(2.79) are rewritten in the form of $[A]\{q\}=\{b\}$,

Surface 1 energy balance,

$$q_{0,1}-(1-\rho_1)q_{i,2}+0q_{o,2}+0q_{i,3}+0q_{o,3}+0q_{i,4}+0q_{o,4}+0q_{i,5}+0q_{o,5}+0q_{o,6}=\rho_1 \qquad (2.80)$$

$$0q_{o,1}-\rho_1 q_{i,2}+q_{o,2}+0q_{i,3}+0q_{o,3}+0q_{i,4}+0q_{o,4}+0q_{i,5}+0q_{o,5}+0q_{o,6}=(1-\rho_1) \qquad (2.81)$$

Layer 1 energy balance,

$$0q_{0,1} - q_{i,2} + 0q_{o,2} + 0q_{i,3} + q_{o,3} + 0q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \tag{2.82}$$

$$0q_{0,1} + 0q_{i,2} + q_{o,2} - q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \tag{2.83}$$

Surface 2 energy balance,

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} - \rho_2 q_{i,3} + q_{0,3} - (1-\rho_2)q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \tag{2.84}$$

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} - (1-\rho_2)q_{i,3} + 0q_{o,3} - \rho_2 q_{i,4} + q_{0,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \tag{2.85}$$

Layer 2 energy balance,

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} - q_{i,4} + 0q_{o,4} + 0q_{i,5} + q_{o,5} + 0q_{o,6} = 0 \tag{2.86}$$

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + q_{o,4} - q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \tag{2.87}$$

Surface 3 energy balance,

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} - \rho_3 q_{i,5} + q_{0,5} + 0q_{o,6} = 0 \tag{2.88}$$

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} - (1-\rho_3)q_{i,5} + 0q_{o,5} + q_{0,6} = 0 \tag{2.89}$$

If the above equations are written with matrix and vector notation,

$$\begin{bmatrix} 1 & -(1-\rho_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_2 & 1 & -(1-\rho_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-\rho_2) & 0 & -\rho_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(1-\rho_3) & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{o,1} \\ q_{i,2} \\ q_{0,2} \\ q_{i,3} \\ q_{o,3} \\ q_{i,4} \\ q_{o,4} \\ q_{i,5} \\ q_{i,5} \\ q_{o,6} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ 1-\rho_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Finally, matrix [A]-vector {b} are given to the subroutine(void Mtx::GaussElim(Vcr& bb)) which is in the Table 8 and the solution is loaded to the vector {b} by the function. Because $q_{o,1}$ is the reflection and $q_{o,6}$ transmission of the system, first and last terms of the vector {b}give the solar radiation properties.

Multi layer system:

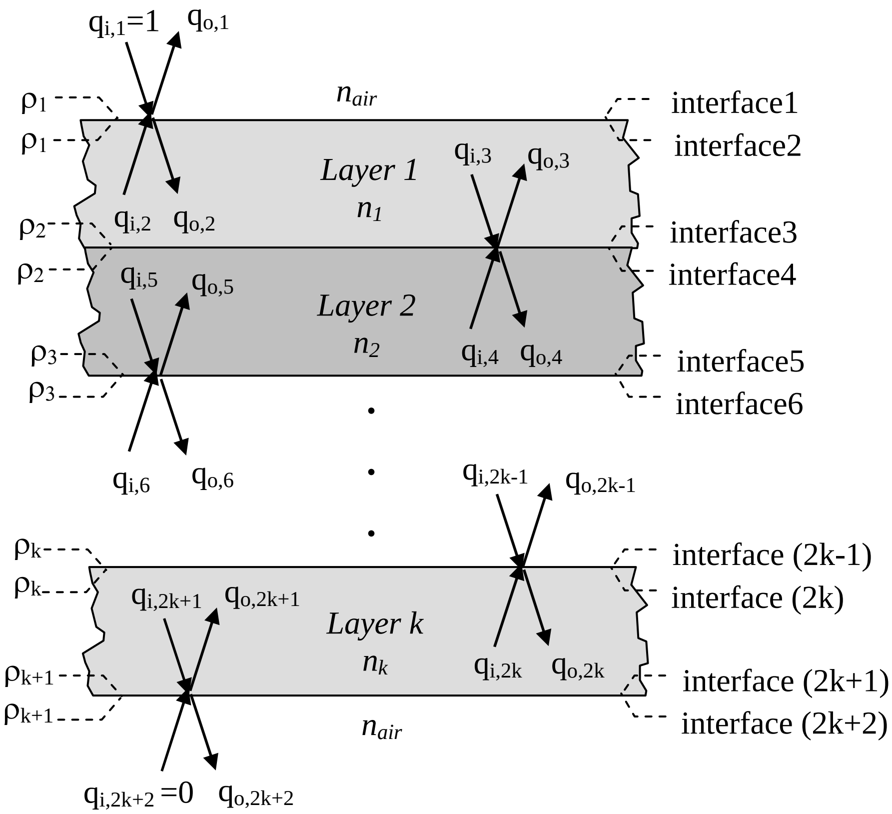


*Figure 51 **:** Net radiation method for non-attenuating multi layer system*

In Figure 51, the surface reflections and notation of outgoing- ingoing fluxes are given. Since there is no any energy loss because of absorption all the wave intensity is transmitted from top surface to the bottom surface and vice versa. For the 'k'th layer system, energy balance equations are written as,

Surface 1 energy balance,

$$q_{0,1} - (1-\rho_1)q_{i,2} + ... + 0q_{i,2k+1} + 0q_{o,2k+1} + 0q_{o,2k+2} = \rho_1 \tag{2.90}$$

$$0q_{o,1} - \rho_1 q_{i,2} + q_{o,2} + ... + 0q_{i,2k+1} + 0q_{o,2k+1} + 0q_{o,2k+2} = (1-\rho_1) \tag{2.91}$$

Layer 1 energy balance,

$$0q_{0,1} - q_{i,2} + 0q_{o,2} + 0q_{i,3} + q_{o,3} + ... + 0q_{i,2k+1} + 0q_{o,2k+1} + 0q_{o,2k+2} = 0 \tag{2.92}$$

$$0q_{0,1} + 0q_{i,2} + q_{o,2} - q_{i,3} + ... + 0q_{i,2k+1} + 0q_{o,2k+1} + 0q_{o,2k+2} = 0 \tag{2.93}$$

Surface 2 energy balance,

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} - \rho_2 q_{i,3} + q_{0,3} - (1-\rho_2)q_{i,4} + ... + 0q_{i,2k+1} + 0q_{o,2k+1} + 0q_{o,2k+2} = 0 \tag{2.94}$$

$$0q_{0,1} + ... - (1-\rho_2)q_{i,3} + 0q_{o,3} - \rho_2 q_{i,4} + ... + 0q_{o,2k+1} + 0q_{o,2k+1} + 0q_{o,2k+2} = 0 \tag{2.95}$$

$\vdots$

Surface k+1 energy balance,

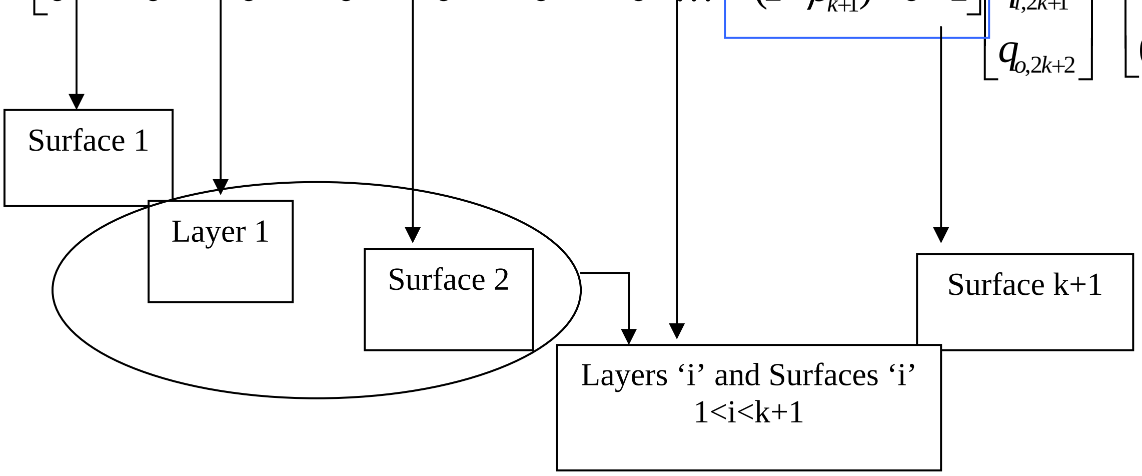
$$0q_{0,1} + \ldots - \rho_{k+1} q_{i,2k+1} + q_{0,2k+1} + 0q_{o,2k+2} = 0 \tag{2.96}$$

$$0q_{0,1} \ldots - (1-\rho_{k+1}) q_{i,2k+1} + 0q_{o,2k+1} + q_{o,2k+2} = 0 \tag{2.97}$$

'4k+2' equations are obtained from the energy balance. The surface reflectance and surface transmission are known. A unit incoming flux($q_{i,1}$=1) to the surface 1 and a zero incoming flux($q_{i,2k+2}$=0) to the surface '2k+2' are assumed. And there are '4k+2' remaining unknowns. If this equation system is solved than solar radiation properties can be obtained. As the solution process again Gauss elimination method explained in the Appendix is used.

If the above equations between (2.90)-(2.97) are written in the form of $[A]\{q\} = \{b\}$,

$$\begin{bmatrix} 1 & -(1-\rho_1) & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & -\rho_1 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_2 & 1 & -(1-\rho_2) & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-\rho_2) & 0 & -\rho_2 & 1 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & -\rho_{k+1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & -(1-\rho_{k+1}) & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{o,1} \\ q_{i,2} \\ q_{0,2} \\ q_{i,3} \\ q_{o,3} \\ q_{i,4} \\ q_{0,4} \\ \vdots \\ q_{i,2k+1} \\ q_{i,2k+1} \\ q_{o,2k+2} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ 1-\rho_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



For the 'k'th layer system, because of the '4k+2' number of unknowns, the size of the matrix A is '4k+2' by '4k+2'. The positions of the layer and surface equations in the matrix are shown above. The part for the layer 1 is repeated exactly for the other layers. And the part for the surface 2 is the same for other surfaces with change of surface reflectance value(ρ_i).

For the solution Gauss elimination method is considered. And by using the subroutine in the Table 8 the problem is solved for the transmission and the reflection of the multilayer system.

2.3.2.2 Attenuating Materials

Again firstly two layered system is discussed than the results is generalized for the multilayer system.

Two layer system:

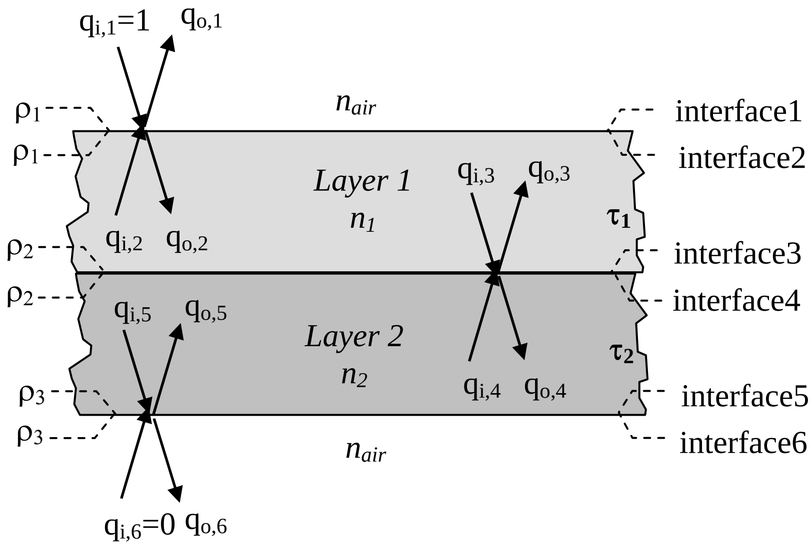


*Figure 52 **:** Net radiation method for attenuating two layer system*

Energy balance equations are written by using the surface reflections and notation of fluxes shown in the Figure 52,

$$q_{0,1} = \rho_1 q_{i,1} + (1-\rho_1)q_{i,2} = \rho_1 + (1-\rho_1)q_{i,2} \tag{2.98}$$

$$q_{0,2} = (1-\rho_1)q_{i,1} + \rho_1 q_{i,2} = (1-\rho_1) + \rho_1 q_{i,2} \tag{2.99}$$

$$q_{0,3} = \rho_2 q_{i,3} + (1-\rho_2)q_{i,4} \tag{2.100}$$

$$q_{0,4} = (1-\rho_2)q_{i,3} + \rho_2 q_{i,4} \tag{2.101}$$

$$q_{0,5} = \rho_3 q_{i,5} + (1-\rho_3)q_{i,6} = \rho_3 q_{i,5} \tag{2.102}$$

$$q_{0,6} = (1-\rho_3)q_{i,5} + \rho_3 q_{i,6} = (1-\rho_3)q_{i,5} \tag{2.103}$$

Attenuating material property is considered in the layer. Since there is energy loss, the transmittance of the layer under the absorption losses,

$$q_{i,2} = q_{o,3}\tau_1 \qquad \text{and} \qquad q_{i,3} = q_{o,2}\tau_1 \tag{2.104}$$

$$q_{i,4} = q_{0,5}\tau_2 \qquad \text{and} \qquad q_{i,5} = q_{0,4}\tau_2 \tag{2.105}$$

Because of unit incoming flux($q_{i,1}$=1) on the surface 1 and a zero incoming flux($q_{i,6}$=0) on the surface 3 there are 10 unknowns and 10 equations. If the above equations are written as below and Gauss elimination method is applied to them the solar properties are obtained

Surface 1 energy balance,

$$q_{0,1} - (1-\rho_1)q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = \rho_1 \qquad (2.106)$$

$$0q_{o,1} - \rho_1 q_{i,2} + q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = (1-\rho_1) \qquad (2.107)$$

Layer 1 energy balance,

$$0q_{0,1} - q_{i,2} + 0q_{o,2} + 0q_{i,3} + \tau_1 q_{o,3} + 0q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \qquad (2.108)$$

$$0q_{0,1} + 0q_{i,2} + \tau_1 q_{o,2} - q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \qquad (2.109)$$

Surface 2 energy balance,

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} - \rho_2 q_{i,3} + q_{0,3} - (1-\rho_2)q_{i,4} + 0q_{o,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \qquad (2.110)$$

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} - (1-\rho_2)q_{i,3} + 0q_{o,3} - \rho_2 q_{i,4} + q_{0,4} + 0q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \qquad (2.111)$$

Layer 2 energy balance,

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} - q_{i,4} + 0q_{o,4} + 0q_{i,5} + \tau_2 q_{o,5} + 0q_{o,6} = 0 \qquad (2.112)$$

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + \tau_2 q_{o,4} - q_{i,5} + 0q_{o,5} + 0q_{o,6} = 0 \qquad (2.113)$$

Surface 3 energy balance,

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} - \rho_3 q_{i,5} + q_{0,5} + 0q_{o,6} = 0 \qquad (2.114)$$

$$0q_{0,1} + 0q_{i,2} + 0q_{o,2} + 0q_{i,3} + 0q_{o,3} + 0q_{i,4} + 0q_{o,4} - (1-\rho_3)q_{i,5} + 0q_{o,5} + q_{0,6} = 0 \qquad (2.115)$$

If the above equations are written with matrix and vector notation, $[A]\{q\} = \{b\}$,

$$\begin{bmatrix} 1 & -(1-\rho_1) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & \tau_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_2 & 1 & -(1-\rho_2) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-\rho_2) & 0 & -\rho_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & \tau_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\rho_3 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -(1-\rho_3) & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{o,1} \\ q_{i,2} \\ q_{0,2} \\ q_{i,3} \\ q_{o,3} \\ q_{i,4} \\ q_{o,4} \\ q_{i,5} \\ q_{i,5} \\ q_{o,6} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ 1-\rho_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

If matrix [A] and vector {b} are compared with those in the section 2.3.2.1 only in the matrix A there are very small differences. The difference comes from the layers energy balance equations. The surface parts does not change. For the solution same subroutine explained in the Table 8 is used. By giving matrix [A]-vector {b} to the subroutine the solar radiation properties is loaded to the first($q_{o,1}$=reflection) and last terms($q_{o,2k+2}$=transmission) of the vector {b}. Absorption is obtained subtraction of the transmission and reflection from unity(1).

Multi layer system:

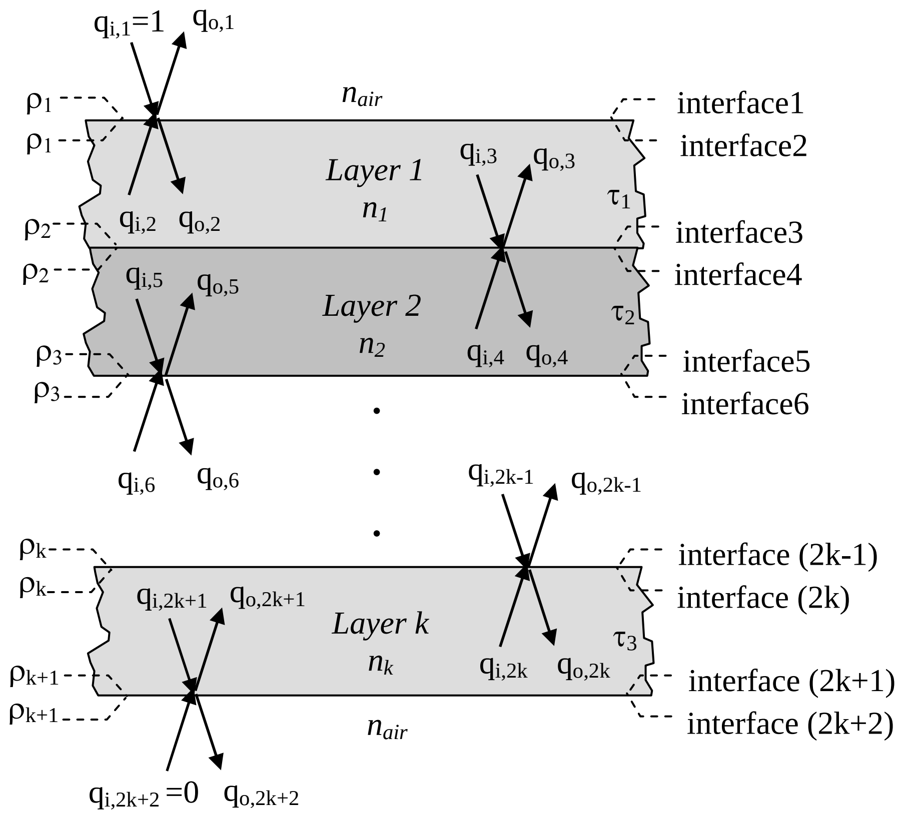


*Figure 53 **:** Net radiation method for attenuating multi layer system*

In Figure 53, the surface reflections, layer absorption coefficient and notation of outgoing-ingoing fluxes are given. For the attenuating 'k'[th] layer system, energy balance equations are written as,

Surface 1 energy balance,

$$q_{0,1}-(1-\rho_1)q_{i,2}+...+0q_{i,2k+1}+0q_{o,2k+1}+0q_{o,2k+2}=\rho_1 \quad (2.116)$$

$$0q_{o,1}-\rho_1 q_{i,2}+q_{o,2}+...+0q_{i,2k+1}+0q_{o,2k+1}+0q_{o,2k+2}=(1-\rho_1) \quad (2.117)$$

Layer 1 energy balance,

$$0q_{0,1}-q_{i,2}+0q_{o,2}+0q_{i,3}+\tau_1 q_{o,3}+...+0q_{i,2k+1}+0q_{o,2k+1}+0q_{o,2k+2}=0 \quad (2.118)$$

$$0q_{0,1}+0q_{i,2}+\tau_1 q_{o,2}-q_{i,3}+...+0q_{i,2k+1}+0q_{o,2k+1}+0q_{o,2k+2}=0 \quad (2.119)$$

Surface 2 energy balance,

$$0q_{0,1}+0q_{i,2}+0q_{o,2}-\rho_2 q_{i,3}+q_{0,3}-(1-\rho_2)q_{i,4}+\ldots+0q_{i,2k+1}+0q_{o,2k+1}+0q_{o,2k+2}=0 \qquad (2.120)$$

$$0q_{0,1}+\ldots-(1-\rho_2)q_{i,3}+0q_{o,3}-\rho_2 q_{i,4}+\ldots+0q_{o,2k+1}+0q_{o,2k+1}+0q_{o,2k+2}=0 \qquad (2.121)$$

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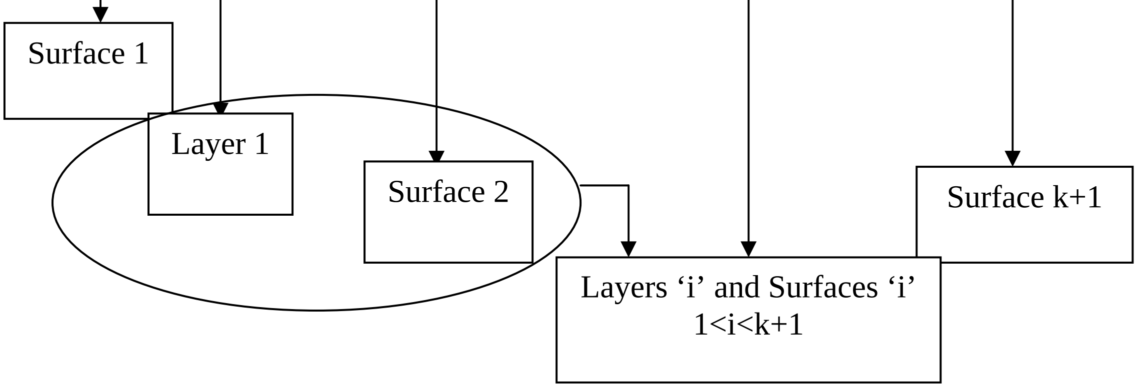
Surface k+1 energy balance,

$$0q_{0,1}+\ldots-\rho_{k+1}q_{i,2k+1}+q_{0,2k+1}+0q_{o,2k+2}=0 \qquad (2.122)$$

$$0q_{0,1}\ldots-(1-\rho_{k+1})q_{i,2k+1}+0q_{o,2k+1}+q_{o,2k+2}=0 \qquad (2.123)$$

Because of unit incoming flux($q_{i,1}=1$) from outside and zero incoming flux($q_{i,2k+2}=0$) from inside there are '4k+2' unknowns with '4k+2' equations. If the above equations between (2.116)-(2.123) are written in the form of $[A]\{q\}=\{b\}$,

$$\begin{bmatrix} 1 & -(1-\rho_1) & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & -\rho_1 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & \tau_1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & \tau_1 & -1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_2 & 1 & -(1-\rho_2) & 0 & \ldots & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-\rho_2) & 0 & -\rho_2 & 1 & \ldots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & -\rho_{k+1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & -(1-\rho_{k+1}) & 0 & 1 \end{bmatrix} \begin{bmatrix} q_{o,1} \\ q_{i,2} \\ q_{0,2} \\ q_{i,3} \\ q_{o,3} \\ q_{i,4} \\ q_{0,4} \\ \vdots \\ q_{i,2k+1} \\ q_{i,2k+1} \\ q_{o,2k+2} \end{bmatrix} = \begin{bmatrix} \rho_1 \\ 1-\rho_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



The 'k'th layer system has '4k+2' number of unknowns, '4k+2' by '4k+2' size of the matrix. It can be very easily seen also that the difference between the non-absorbing condition is due to absorption losses in the layer part. The other parts are exactly the same. Apart from that part of the layer 1 and surface 2 are repeated for other layer and surfaces with change of surface reflectance value(ρ_i) and absorption coefficient(τ_i). Matrix A and vector b are solved again by using Gauss elimination method to obtain reflection, absorption and transmission values.

2.3.3 Averaged Properties

Spectral and polarization average are explained in the single layer part(Section 2.1.3). Everything is the same also here. If any problem please look at that section.

2.3.4 Some Examples

It is very hard to solve the multilayer system with hand so software tool is used. For that reason only one example is given illustrating vary complicated condition.

Example 14:

Aim of the example: 3 layer attenuating system is illustrated.

Question: Compute the reflection, transmission and absorption of the attenuating layers below for the given wavelength spectrum(see Appendix) at the incidence angle of 0 degree. And compute the spectral averaged values for the angles between 0-90.

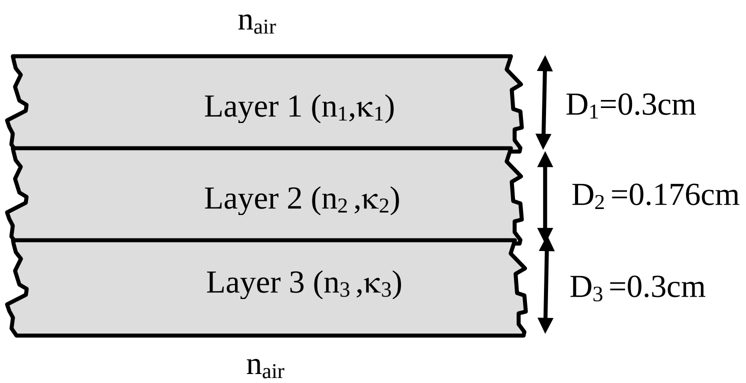
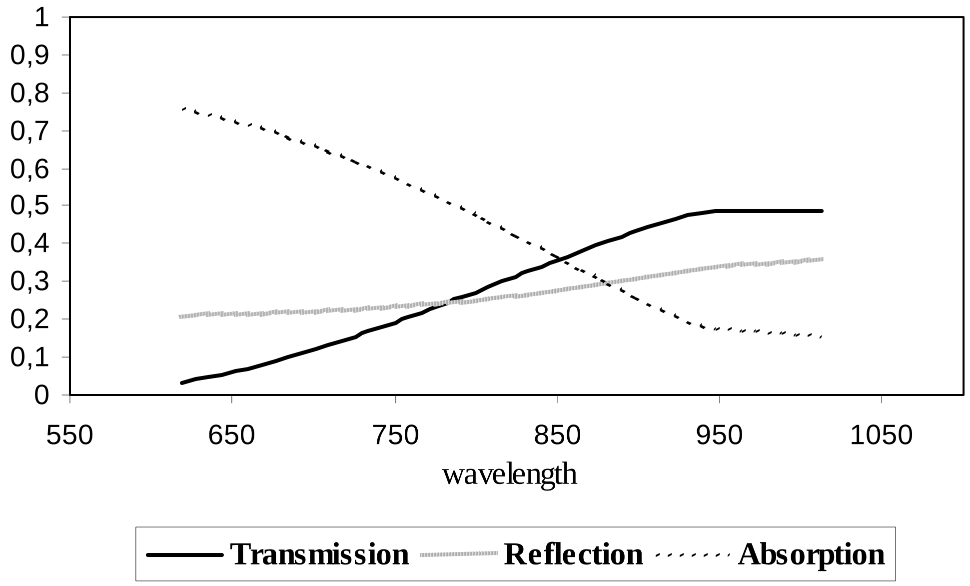


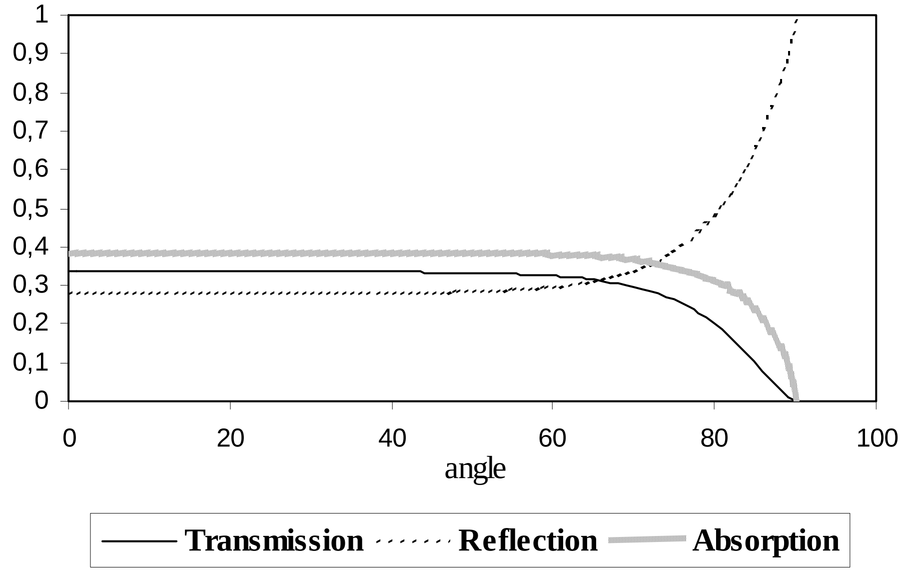
*Figure 54 **:** Properties of window for Example 14*

The problem is solved by using software tool and the results are reached in the CD-Excel sheets- Example 14. In Graph 14 the transmission, reflection, and absorption vs. wavelength is seen,



*Graph 14 **:** Transmission, reflection, absorption vs. wavelength for attenuating 3 layer system at the normal incidence in Example 14*

If above graph is obtained for all angle of incidence between 0 and 90 degree then averaged over the wavelength with the method explained in the Section 2.1.3.1, solar radiation properties vs. wavelength graph(CD-Excel sheets- Example 14) become like below,



*Graph 15 **:** Transmission, reflection, absorption vs. angle of incidence between 0 to 90 degree for attenuating 3 layer system at the normal incidence*

2.4 Partially Transmitting Multi-Layer with Thickness D<λ(with Wave Interference Effect)

2.4.1 Non-Attenuating Multi Layer

Firstly two layered system is discussed than the results is generalized for the multilayer system. Assumption of non-attenuation material property causes zero absorption coefficient .

Two layer system:

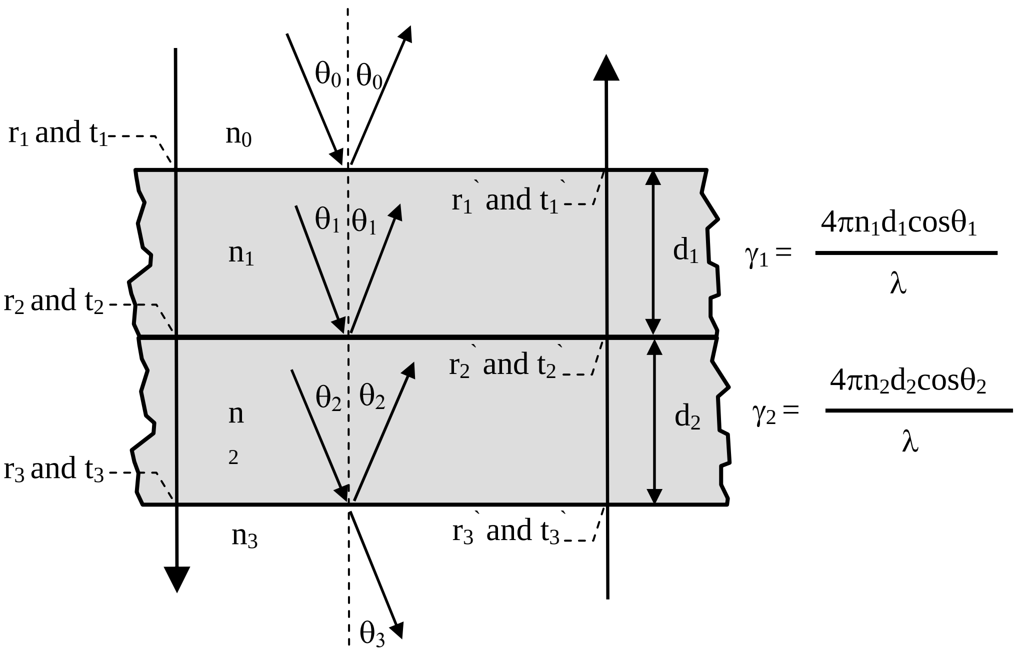


Figure 55 ***:*** *Fresnel coefficients and phase differences of two film layers[3]*

In view of the growth in importance of double-layer films, the results for the reflection, transmission and absorption coefficients are given for the system. The system is as shown in Figure 55. There are several ways in which the result for two films may be obtained.[3]

One of the method is given by P.Rouard[3]. The problem is treated in two stages. *In first part* the surface Fresnel coefficients (r_2, t_2, r_3, t_3) are obtained for the film (n_2) by using the Eq. (1.109)-(1.112) then substrate system transmission and reflection coefficients are calculated using the expressions of single layer(see the Eq. (2.36) and (2.37)). Thus the Fresnel reflection coefficient for the substrate system(n_2) is given by,

$$R_{(n_2)} = \frac{r_2 + r_3 e^{-i\gamma_2}}{1 + r_2 r_3 e^{-i\gamma_2}} \tag{2.124}$$

and the transmission coefficient is given by for the layer(n_2):

$$T_{(n_2)} = \frac{t_2 t_3 e^{-i\gamma_2/2}}{1 + r_2 r_3 e^{-i\gamma_2}} \tag{2.125}$$

where the r_i's and t_i's are the Fresnel coefficient for two surfaces of substrate system as defined by Eq.(1.109)-(1.112).

In second part, the n1 layer is considered as covering a surface. The reflection and transmission coefficients are given by Eq. (2.36) and (2.37) where r_2 and t_2 must be replaced by the coefficients given in first part,

$$R_{sys} = \frac{r_1 + R_{(n_2)} e^{-i\gamma_1}}{1 + r_1 R_{(n_2)} e^{-i\gamma_1}} \tag{2.126}$$

$$T_{sys} = \frac{t_1 T_{(n_2)} e^{-i\gamma_1/2}}{1 + r_1 R_{(n_2)} e^{-i\gamma_1}} \tag{2.127}$$

As a results if equations between (2.124)-(2.127) are combined, the double layer system coefficients are obtained like,

$$R_{sys} = \frac{r_1 + r_2 e^{-i\gamma_1} + r_3 e^{-i(\gamma_1+\gamma_2)} + r_1 r_2 r_3 e^{-i\gamma_2}}{1 + r_1 r_2 e^{-i\gamma_1} + r_1 r_3 e^{-i(\gamma_1+\gamma_2)} + r_2 r_3 e^{-i\gamma_2}} \tag{2.128}$$

$$T_{sys} = \frac{t_1 t_2 t_3 e^{-i(\gamma_1+\gamma_2)/2}}{1 + r_1 r_2 e^{-i\gamma_1} + r_1 r_3 e^{-i(\gamma_1+\gamma_2)} + r_2 r_3 e^{-i\gamma_2}} \tag{2.129}$$

Now the extension to a system of multiple layer is considered.

Multi layer system:

Methods by which the derivation of the reflection coefficient for a single layer can be extended to the case of any number of layers. Since a single film bounded by two surfaces possesses an effective reflection coefficient and accompanying phase change, then such a film can be replaced by a single surface with these properties. There are two possible approaches. One of the approach starts with the film next to the supporting substrate and works step by step through the intervening layers to the top of the system. Second method starts with the top layer and moves downwards towards the substrate. The expression for reflectivity of the system are slightly more tractable when the first approach as in Figure 56 is used.[3]

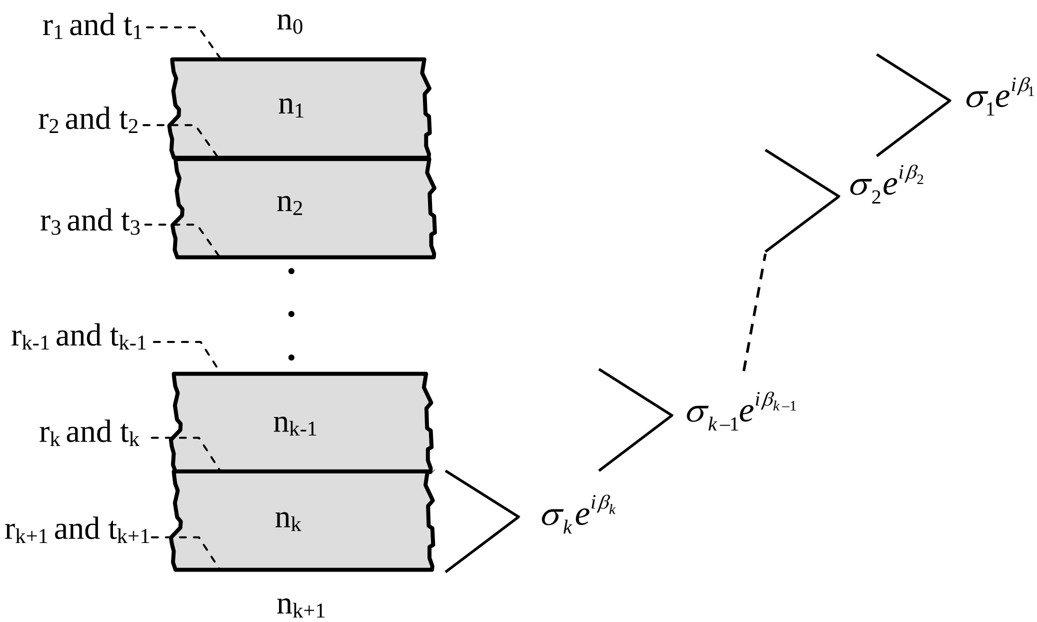


*Figure 56 **:** Bottom to top scheme for non-absorbing multiple layer[3]*

First, the amplitude and phase of the light reflected by the lowest film are computed. Writing σ_k for the real amplitude and β_k for the phase of k[th] layer,

$$\sigma_k e^{i\beta_k} = \frac{r_k + r_{k+1}e^{-i\gamma_k}}{1 + r_k r_{k+1}e^{-i\gamma_k}} \qquad (2.130)$$

This is the effective Fresnel coefficient for the layer n_k, and is inserted into the corresponding expression for the Fresnel coefficient for the layer n_{k-1} whose Fresnel coefficient is,

$$\sigma_{k-1} e^{i\beta_{k-1}} = \frac{r_{k-1} + \sigma_k e^{i\beta_k} e^{-i\gamma_{k-1}}}{1 + r_{k-1}\sigma_k e^{i\beta_k} e^{-i\gamma_{k-1}}} \qquad (2.131)$$

And it is continued until the first layer, finally system reflection amplitude is obtained,

$$R_{sys} = \sigma_1 e^{i\beta_1} = \frac{r_1 + \sigma_2 e^{i\beta_2} e^{-i\gamma_1}}{1 + r_1\sigma_2 e^{i\beta_2} e^{-i\gamma_1}} \qquad (2.132)$$

If this procedure is applied to the transmission from the last layer to the first layer assuming the ν_k for the real amplitude and ψ_k for the phase of k[th] layer, the amplitude of the system transmission is,

$$\vartheta_k e^{i\psi_k} = \frac{t_k t_{k+1} e^{-i\gamma_k/2}}{1 + r_k r_{k+1} e^{-i\gamma_k}}$$

$$\vdots \qquad (2.133)$$

$$T_{sys} = \vartheta_1 e^{i\psi_1} = \frac{t_1 \vartheta_2 e^{i\psi_2} e^{-i\gamma_1/2}}{1 + r_1\sigma_2 e^{i\beta_2} e^{-i\gamma_1}}$$

As explained in the Section (1.1.3) and Section (2.2.1) the wave energy depends on $|\mathrm{E}|^2$. According to the Poynting vector(Eq.(1.46)) the reflectivity and transmission for the system are

$$R = |R_{sys}|^2 = \frac{n_0}{n_0} R_{sys} R_{sys}^* = R_{sys} R_{sys}^* \qquad (2.134)$$

$$T = |T_{sys}|^2 = \frac{n_{k+1}}{n_0} T_{sys} T_{sys}^* \qquad (2.135)$$

where R_{sys}^* is the complex conjugate of R_{sys} and T_{sys}^* is the complex conjugate of T_{sys}.

2.4.2 Attenuating Multi Layer

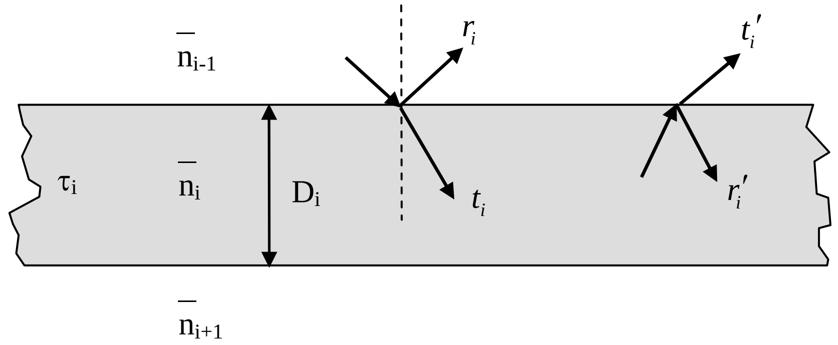


Figure 57 : Layer properties and Fresnel's coefficients of i^{th} attenuating layer in the system

If the layers are absorbing energy, the amplitudes of the successively reflected and transmitted beams in terms of the Fresnel's coefficients which are complex number are given in the Section (1.2.4). Fresnel's coefficients for propagation are seen in the Figure 57. The phase difference for the attenuating material is discussed in the Section 2.2.2 and it is obtained as,

$$\gamma_i = 4\pi(\mathrm{Re}(\bar{n}_i))D_i / [\lambda_0 \,\mathrm{Re}(\cos\chi)] \tag{2.136}$$

After remember the surface Fresnel coefficients and phase difference for the absorbing layers now the system solar radiation coefficients are calculated by using the same method that is discussed previous section. That approach starts with the film next to the supporting substrate and works step by step through the intervening layers to the top of the system. The expression for reflectivity of the system is obtained with this approach as in Figure 56 ,

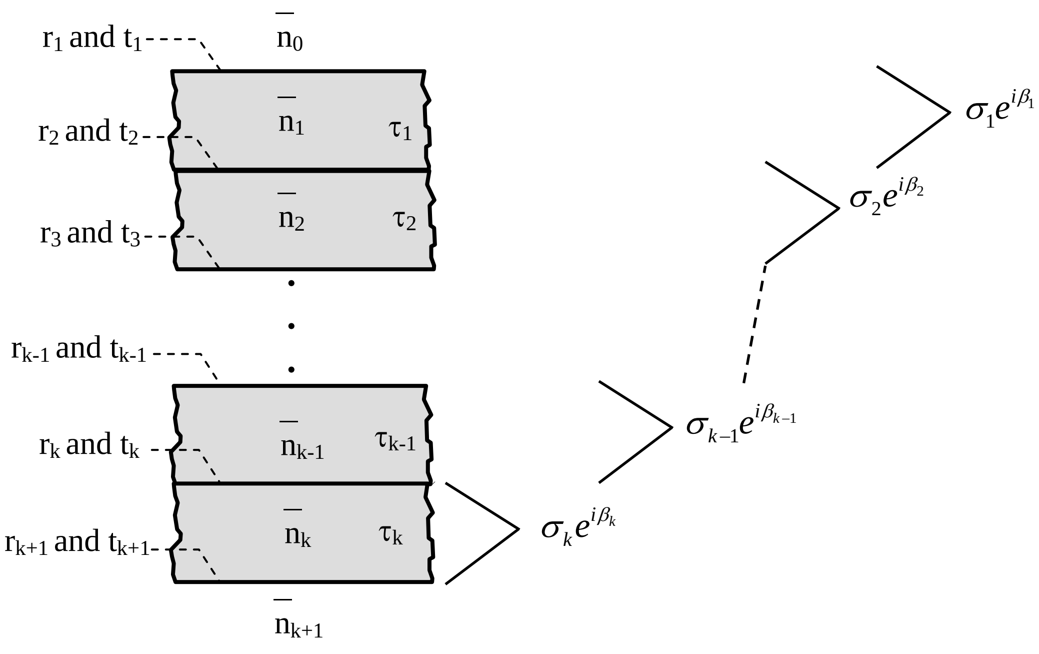


Figure 58 : Bottom to top scheme for absorbing multiple layer

First, the amplitude and phase of the light reflected by the lowest film are computed,

$$\sigma_k e^{i\beta_k} = \frac{r_k + r_{k+1}\tau_k^{\ 2} e^{-i\gamma_k}}{1 + r_k r_{k+1}\tau_k^{\ 2} e^{-i\gamma_k}} \tag{2.137}$$

This equation is inserted into the corresponding expression for the Fresnel coefficient for the layer n_{k-1} whose Fresnel coefficient is,

$$\sigma_{k-1} e^{i\beta_{k-1}} = \frac{r_{k-1} + \sigma_k e^{i\beta_k}\tau_{k-1}^{\ 2} e^{-i\gamma_{k-1}}}{1 + r_{k-1}\sigma_k e^{i\beta_k}\tau_{k-1}^{\ 2} e^{-i\gamma_{k-1}}} \tag{2.138}$$

Finally system reflection amplitude is obtained,

$$R_{sys} = \sigma_1 e^{i\beta_1} = \frac{r_1 + \sigma_2 e^{i\beta_2}\tau_1^{\ 2} e^{-i\gamma_1}}{1 + r_1\sigma_2 e^{i\beta_2}\tau_1^{\ 2} e^{-i\gamma_1}} \tag{2.139}$$

If this procedure is applied to the transmission from the last layer to the first layer assuming the ν_k for the real amplitude and ψ_k for the phase of k[th] layer , the amplitude of the system transmission is,

$$\vartheta_k e^{i\psi_k} = \frac{t_k t_{k+1}\tau_k e^{-i\gamma_k/2}}{1 + r_k r_{k+1}\tau_k^{\ 2} e^{-i\gamma_k}}$$

$$\vdots \tag{2.140}$$

$$T_{sys} = \vartheta_1 e^{i\psi_1} = \frac{r_1\vartheta_2 e^{i\psi_2}\tau_1 e^{-i\gamma_1/2}}{1 + r_1\sigma_2 e^{i\beta_2}\tau_1^{\ 2} e^{-i\gamma_1}}$$

As explained in the Section (1.1.3) and Section (2.2.1) the wave energy depends on $\left|\mathrm{E}\right|^2$. According to the Poynting vector(Eq.(1.46)) the reflectivity and transmission for the system are

$$R = \left|R_{sys}\right|^2 = \frac{n_0}{n_0} R_{sys} R_{sys}^{\ *} = R_{sys} R_{sys}^{\ *} \tag{2.141}$$

$$T = \left|T_{sys}\right|^2 = \frac{n_{k+1}}{n_0} T_{sys} T_{sys}^{\ *} \tag{2.142}$$

where $R_{sys}^{\ *}$ is the complex conjugate of R_{sys} and $T_{sys}^{\ *}$ is the complex conjugate of T_{sys}.

2.4.3 Averaged Properties

The solar radiation properties for the system discussed above are all implicit functions of wavelength, incident angle and polarization. To obtain the spectral and polarization averaged values the same methods which are explained in the Section 2.1.3 are used also in that section. For that reason they are not explained in detail so if any problem please look at that section.

2.5 Comparison the results obtained with and without interference effect consideration

In that section the discussion is solving the very thick layers assuming the interference effect and comparing the solution with the method where the interference effect is neglected. To explain that subject two examples are used. For simplicity firstly, one layer system with non-absorbing material theory is considered. Than attenuation property is added and the results are discussed.

Example 15:

Aim of the example: Consideration of wave interference effect in the thick layers($D>\lambda$)

Question: Compute the reflection, transmission and absorption of the non-attenuating layer below for the given wavelength spectrum at the incidence angle of 0 degree with and without interference effect. And compare the results.(see the Appendix for the data set)

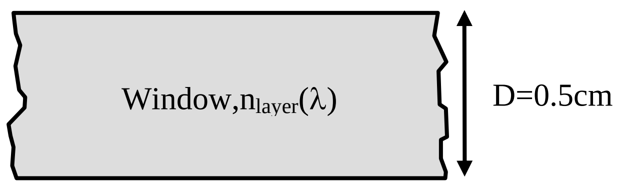
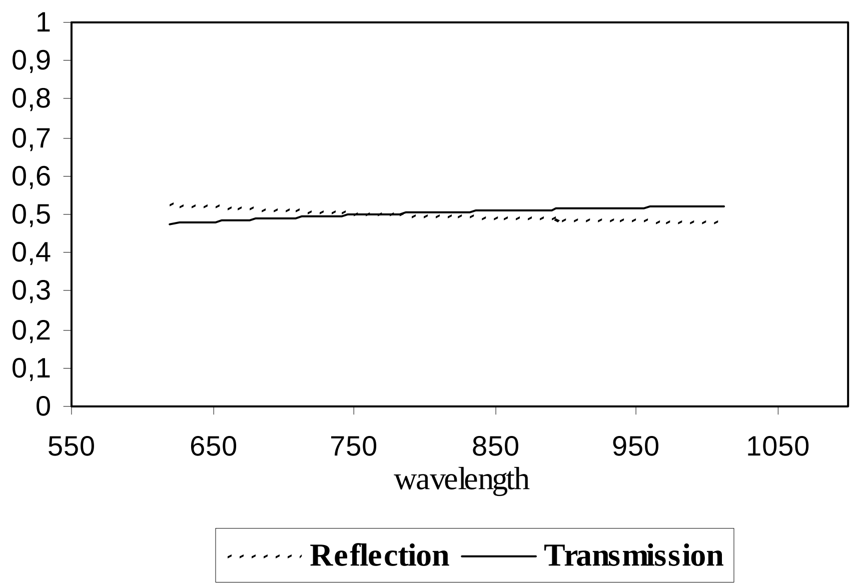


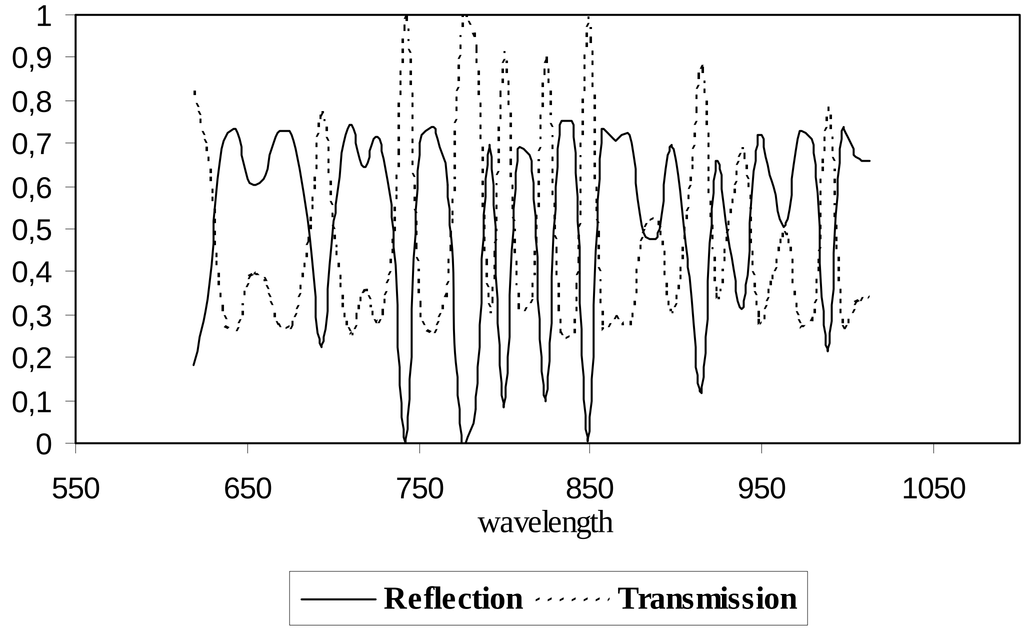
*Figure 59 **:** Properties of window for the example in Section 2.5*

The thickness of the layer is thicker according to the wavelength of the light(see appendix). So the interference effect can be neglected and the problem is solved with the explained procedure in the Section 2.1.2.1. By using the software tool below graph is obtained for the solar radiation coefficients(see also the CD-Excel sheets-Example 15 for the results details),



*Graph 16 **:** Transmission and reflection vs. wavelength for non-attenuating single layer at the normal incidence neglecting the wave interference effect in* Example 15

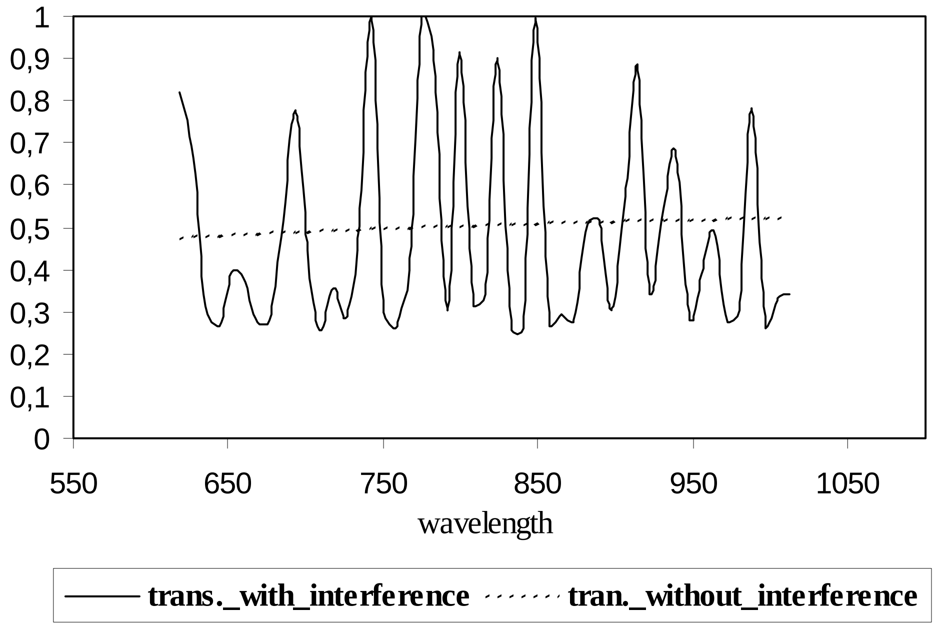
Now even the thickness of the layer wave interference effect is not neglected. The theory explained in the Section 2.2.1 for the condition D<λ is now applied to the thick layer. By using again the software tool the problem is solved(see the CD-Excel sheets-Example 15),



*Graph 17 **:** Transmission and reflection vs. wavelength for non-attenuating single layer at the normal incidence considering the wave interference effect in Example 15*

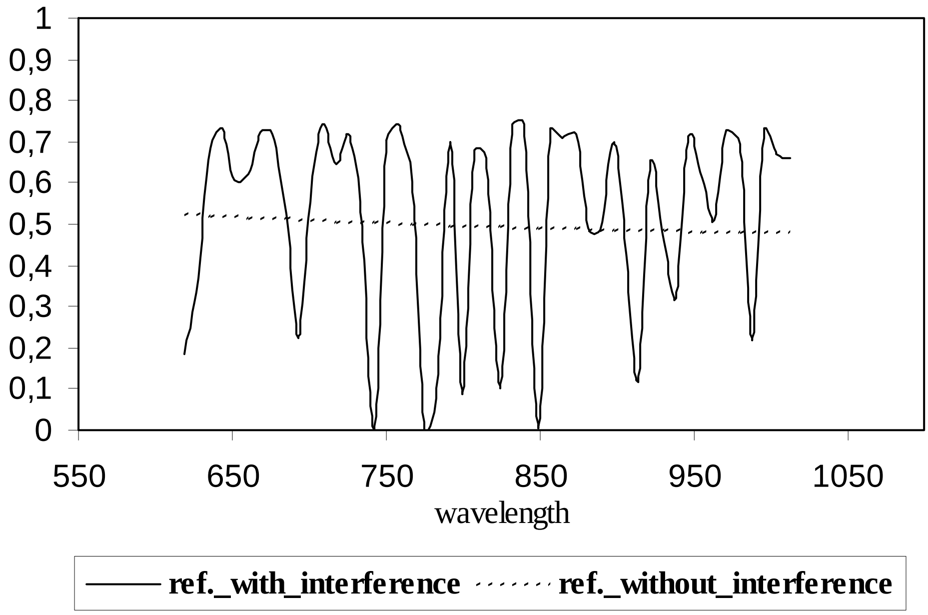
As it can be seen in the Graph 17, because of the thickness of the layer at the more wavelengths zero and minimum reflection takes place. So this oscillatoric graph is obtained. At that point in order to be able to compare the two method both graph will be drawn in common one.

Transmissions and reflections are compared,



*Graph 18 **:** Comparison of the transmission for non-attenuating material*

If the transmissions are integrated over the wavelength by using the Eq.(2.28), transmission obtained neglecting the wave interference effect is 0.51 and transmission obtained considering the wave interference effect is 0.50. As it can be seen that they are almost same. So the method that is consist of the wave interference effect can be used for all condition. But here the problematic part is this method is much more complex and needs additional inputs. All the things also valid for the reflection.



Graph 19 : Comparison of the reflection for non-attenuating material

Example 16:

Aim of the example: Consideration of wave interference effect in the thick layers(D>λ) for the absorbing layer.

Question: Compute the reflection, transmission and absorption of the attenuating layer below for the given wavelength spectrum at the incidence angle of 0 degree with and without interference effect. And compare the results.(see the Appendix for the data set)

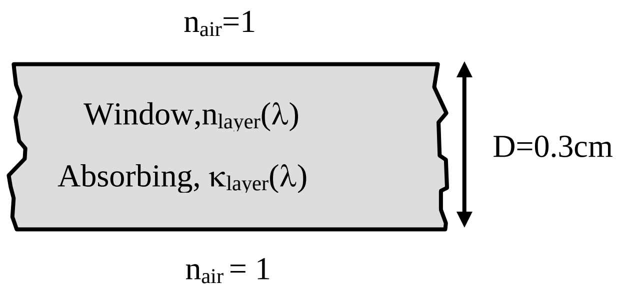
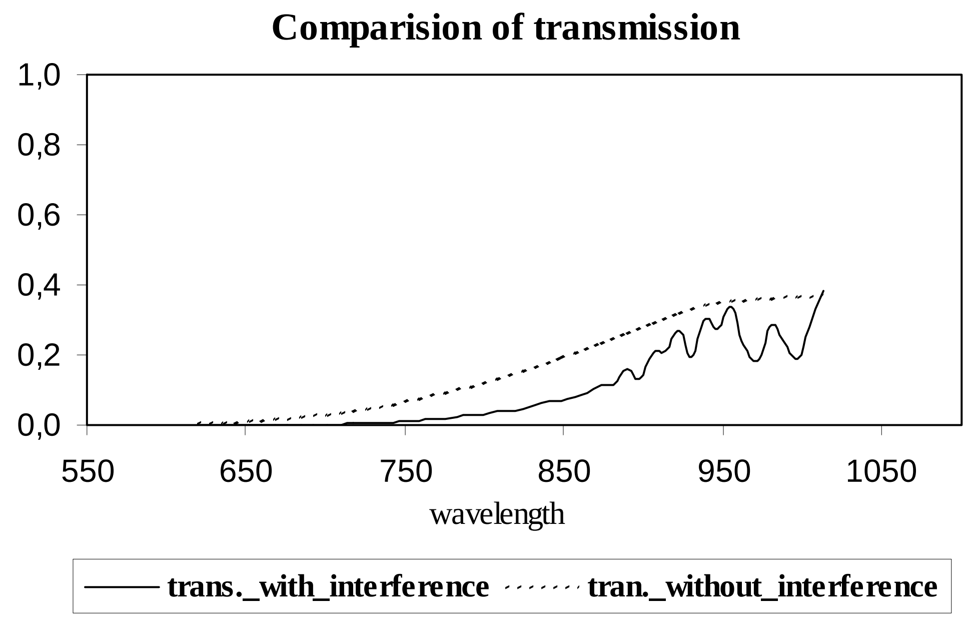


Figure 60 : Properties of window for the example in Section 2.5

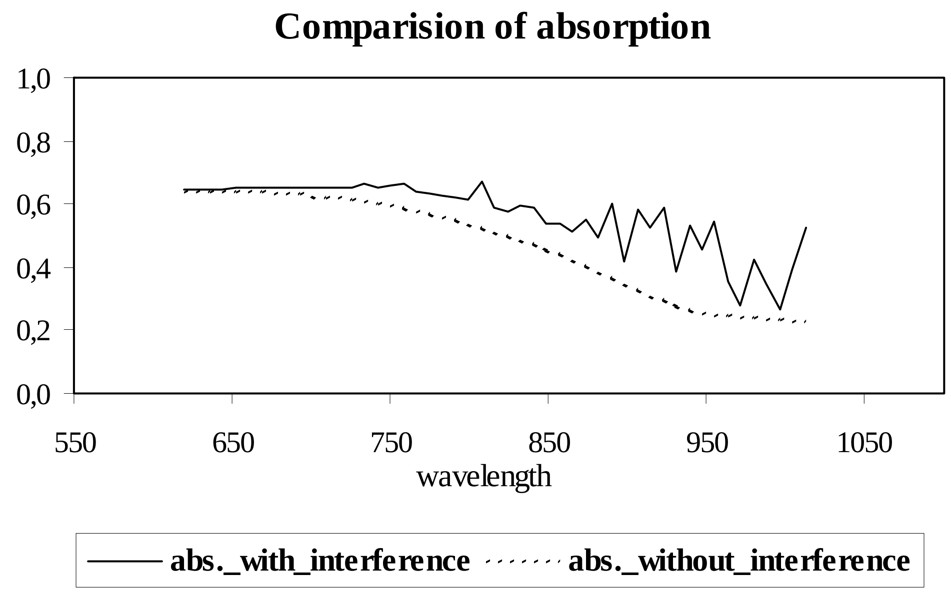
In that example, like previous one , even the thickness of the layer is not thin, the problem is solved with the both method and the result are obtained. Here, only the comparison graphs are shown. The others and the details of below results can be found in the CD-Excel sheets-Example 16.

Transmissions are compared,



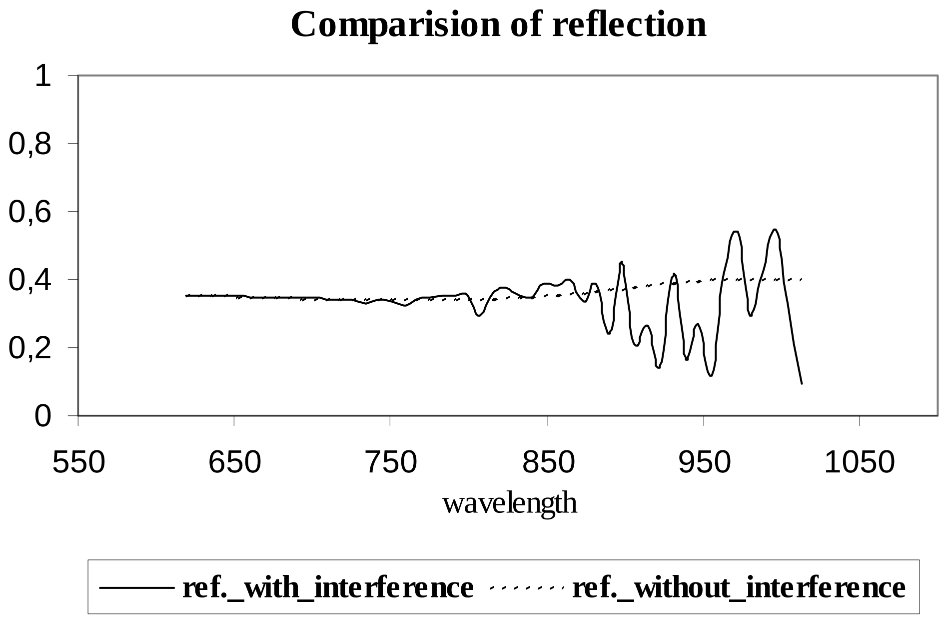
*Graph 20 **:** Comparison of the transmission for attenuating material*

If the transmissions are integrated over the wavelength by using the Equation (2.28), transmission obtained neglecting the wave interference effect is 0.2 and transmission obtained considering the wave interference effect is 0.12. There is small difference between them. When the wave interference is not neglected transmission reduces. The reason is the waves which are in the phase shift also absorbed during the travel in the layer. This additional absorbing decreases the transmission and increases the absorption. Below this rise in the attenuation is seen at the below graph,



*Graph 21 **:** Comparison of the absorption for attenuating material*

If the absorption coefficients are integrated over the wavelength, attenuation increases from 0.44 to 0.53 when the wave interference effect is considered. And the reflection is almost same for both method,



Graph 22 **:** *Comparison of the reflection for attenuating material*

Software Tool

A software tool is constructed to calculate the glazing system solar radiation coefficients. For this work C++ compiler and Qt are used. During the programming process, the information that is explained in the theory part is implemented. And here the user interface part, explanation of how this tool can be used, is explained.

As in the Figure 61, the program user interface consists of 4 parts,

- Theory
- Input
- Calculation
- Configuration

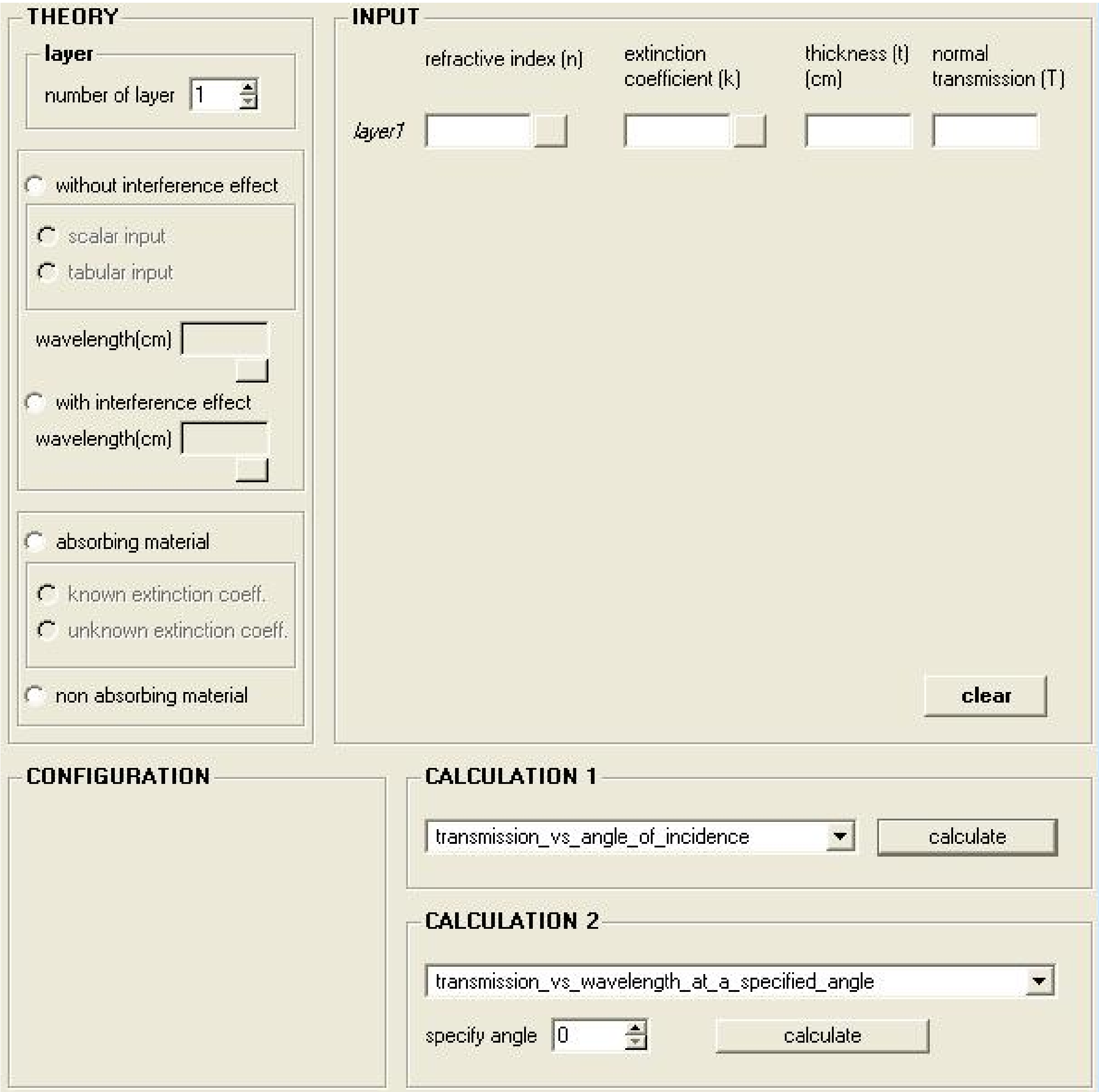


Figure 61 : Program user interface when it is started

THEORY:

In this part, according to your glazing system the appropriate theory is given to the program. Number of layers, wave interference effect and wave attenuation in the material are decided and wavelength spectrum is uploaded to the program.

Number of layer:
First of all, the number of layer in the system is chosen between 1 and 8.

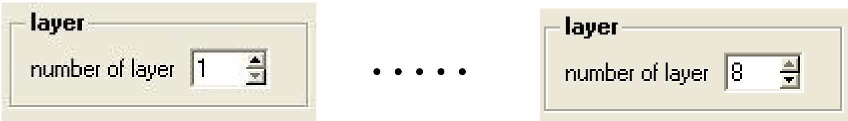


*Figure 62 **:** With the spin box number of layer is decided*

Wave interference effect:
Then, user has to decide wave interference effect in the system. If all the layers in the glazing are thick enough with respect to the wavelength, there appears two choices(Figure 63). These are scalar or tabular input.

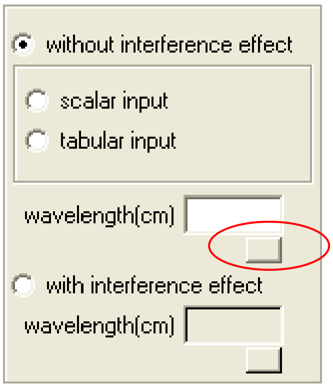


*Figure 63 **:** "without interference effect" is chosen*

Now the user decides one of it according to the inputs. If the inputs are depend on the wavelength than tabular input is clicked. But if the inputs have already integrated over the wavelength and there are averaged inputs then tabular input is clicked. After that wavelength information is given. If wavelength spectrum is known then by clicking the button in the red box(see the Figure 63) a file consists of wavelength spectrum can be upload to the system. Otherwise the value is written by hand directly.

If the thickness of the even one of the layer in the system is thin with respect to wavelength 'with wave interference' is chosen. (see the Figure 64)

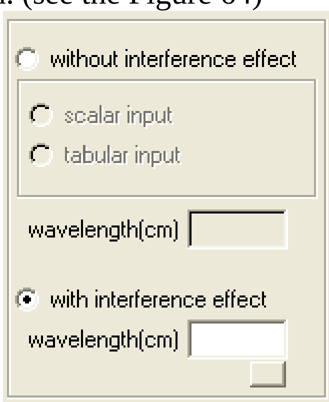


Figure 64 : "with interference effect" is chosen

Wave attenuation through the material:
If wave does attenuate as it travels in the material absorbing material is clicked. And the user has to give another decision according to the type of information. As explained in the Section 2.1.4 for some materials extinction coefficients can not be known. So to find the absorption coefficient, normal transmission of the layers is used. According to the information that the user has one of it is chosen(see the Figure 65).

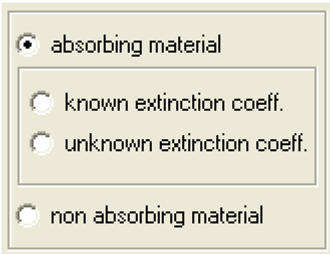


Figure 65 : "absorbing material" is chosen

If material does not absorb any wave energy, non-absorbing material is clicked and there is no need of extinction coefficient any more. (see the Figure 66).

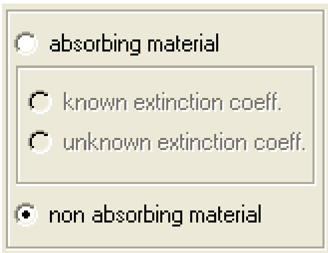


Figure 66 : "non-absorbing material" is chosen

INPUT:

In this part, the user gives datas according to the theory that is given previously. As it can be seen in the Figure 61 the inputs are refraction index, extinction coefficient, thickness of the layer and normal transmission of the layer. But of course all of them are not given at the same time. When the theory part is finished the inputs that should be given to the tool are enabled automatically and the others disabled. Also number of layers are increased or decreased with the chosen number which is given in theory part.

Figure 61 is the picture when the program is started. Then after according to your system properties which are given in the theory is change automatically. For example assume that our glazing consists of 3 layer. The films are thick enough with respect to the wavelength and wave attenuates through the material. Also refractive indexes and extinction coefficients are known at the wavelength spectrum. Then the input part is seen like in the Figure 67,

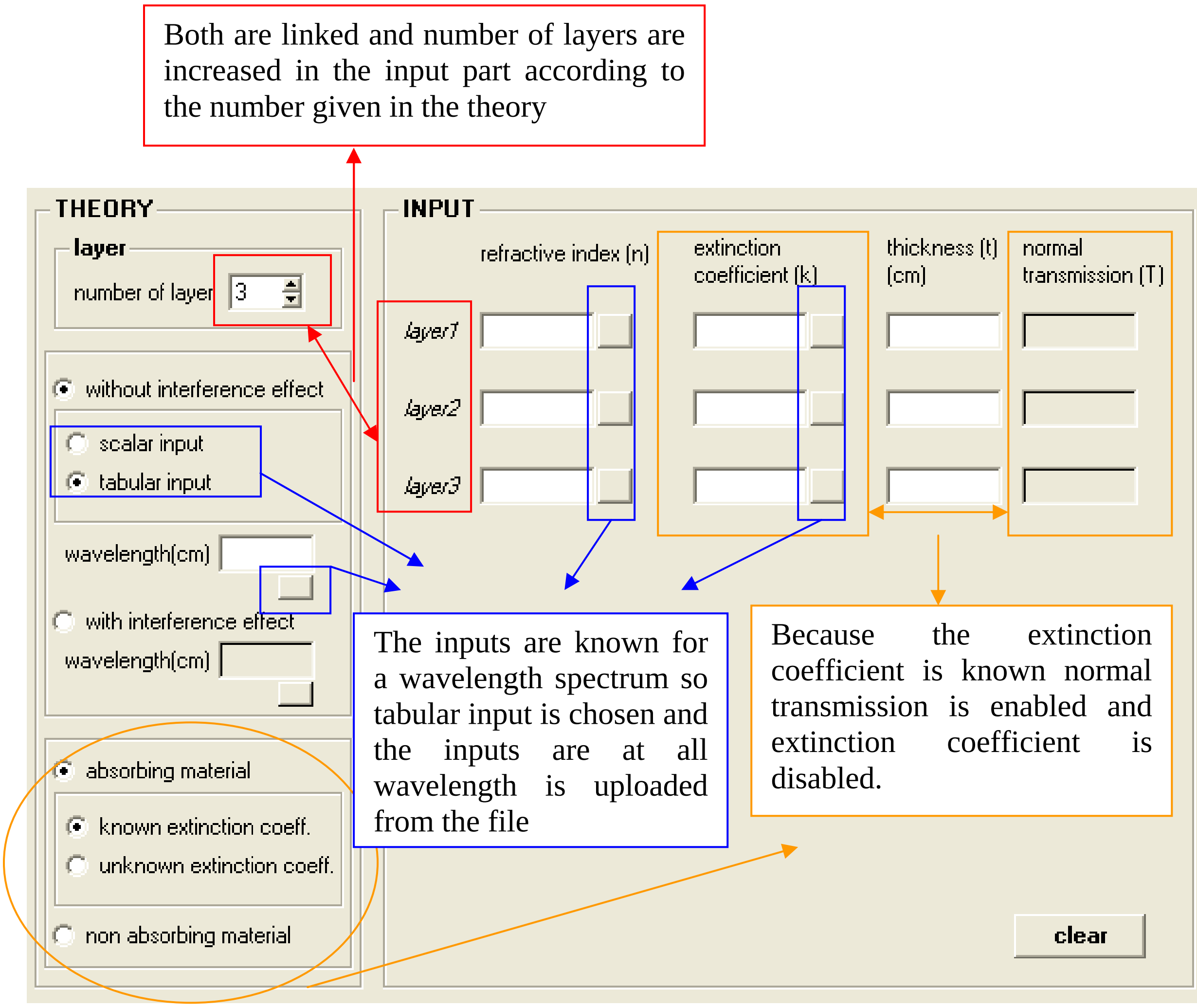


Figure 67 : An example of relation between theory and input parts

CALCULATION:

After theory and inputs are concluded different types of results can be obtained from the tool. For that purpose there are two kind of calculation as it can be seen in the Figure 61.

Calculation 1

This is based on the angles. Transmission or reflection values, this is chosen as in the Figure 68, are given at the angles between 1-90 degrees with the increment of 1 degree.

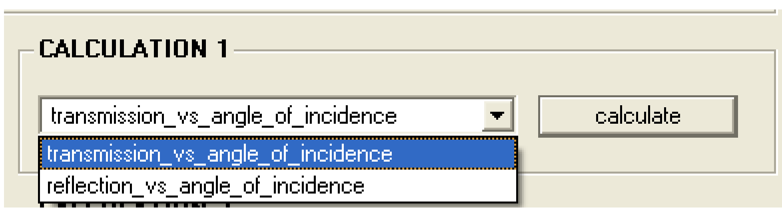


Figure 68 : Calculation 1

Calculation 2
Transmission or reflection values are given at the all wavelength that are uploaded in the theory part. But first of all incident angle must be decided then the result is calculated for reflection or transmission(see Figure 69)

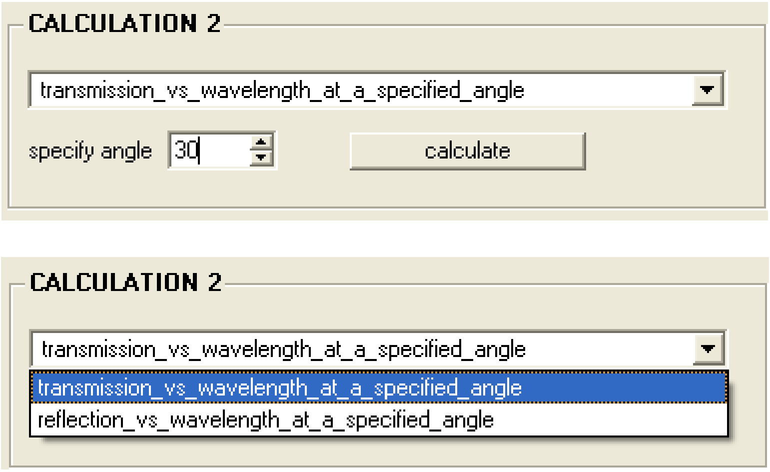


Figure 69 **:** *Calculation 2: first angle of incidence is decided than needed type of the solar radiation coefficient is chosen*

Conclusion

In the thesis mathematical models for glass transmission have been explained and discussions are extended either theoretical or practical problem solutions. The two objective aims, which are finding the solar radiation coefficients and creating a software tool to get the results in practical usage, were reached.

As mentioned before firstly surface transmission and reflection values were investigated by using Maxwell's equations that include electric and magnetic fields(electromagnetic wave theory). The surface solar radiation coefficients were calculated from their optical and electrical properties. The relations between radiative, optical and electrical properties are developed by considering wave propagation in a medium and the interaction when an electromagnetic wave travelling through one medium is incidence on the surface of another medium. Boundary conditions were applied to the polarized plane electric and magnetic fields of the waves which are perpendicular each other and the ratios of wave amplitudes are reached(e.g. $E_{M,r} / E_{M,i}$). The energy carried by a wave is proportional to the square of the wave amplitude. So squaring ratio of amplitudes gives surface solar radiation coefficients. The analysis was done for an ideal interaction, means for optically smooth and clean surfaces, between the incident wave and the surface. It was seen that the factors that effecting the results are refractive index(n) and angle of incidence(θ). Firstly refraction angles are computed by using Snell's law. Then with the result of Snell's law and refractive index surface coefficients were computed. At that point there are two phenomena which are very important in the analysis. These are wave attenuation and wave interference effect. If the material absorbs some energy than the refraction index became complex number like $n - i\kappa$. Here κ is the extinction coefficient. Wave interference effect, which results when two waves are travelling through a medium and meet up at the same location, came into picture when the thickness of the layer is comparable with the wavelength of the light. When two wave meet at the same point 180° phase shift in the wave is introduced which affects the reflection and transmission values. As a result these two phenomena cause complexities during the solution of the problem.

After surface discussion inside of the glazing material was considered. As the wave travels through the material some of the energy is absorbed by the film. How much amount of energy is transmitted along the travelling distance of light(x) is decided by extinction coefficient and wavelength of the light in the vacuum.

Enclosures can have windows that are partially transparent to radiation. A window can be of a single material, or it can have one or more transmitting layers. So to reach one of our goal the system transmission, reflection and absorption mathematical models were discussed. Two methods were introduced. These are,

- Ray tracing method
- Net radiation method

Both method gives the same results for the simple conditions. But as explained in the theory net radiation method is more functional. Because modern heat transfer calculation method can be applied to complicated situations easily with respect to the ray tracing method. The complexity originates from wave attenuation and wave interference effects. So in the software tool the results of net radiation method was basically used.

One of the thing that must be mentioned is the discussion of the wave interference also for thick layers with respect to the wavelength. This subject was explained with the examples and it was concluded that the method that is consist of the wave interference effect can be used for all condition with additional inputs and additional effort.

The solar radiation properties for the system discussed above are all implicit functions of wavelength, incident angle and polarization. As mentioned above the electric and magnetic fields are polarized into parallel and perpendicular components in the mediums. And they are averaged by using arithmetic mean. The optical indices(refraction index-extinction coefficient are also strongly dependent on wavelength. Having calculated the properties at all desired wavelengths, average properties for the glazing system was calculated using the general equation (2.28). For solar radiation the source is sun and the results are weighting with the sun radiation values which are given in the Appendix.

Finally, under the light of theoretical knowledge software tool was constructed for practical usage with the C++ compiler and Qt. The tool can compute transmission and reflection values for every condition that the user needs when the suitable theory is chosen and required inputs are given.

Appendix

In the Table 9, there are layers data which are used in the examples. According to the question if the layer is non-absorbing then extinction coefficients are not considered.

*Table 9 **:** Data set for the examples*

Wavelength(λ)		**Refractive Index(n)**	**Extinction Coefficient(κ)**
wavelength(nm)	*wavelength(cm)*		
619.02	0.000061902	3.9561	0.00007749
627.22	0.000062722	3.94	7.39888E-05
635.42	0.000063542	3.9245	7.06782E-05
643.62	0.000064362	3.9095	0.000067545
651.82	0.000065182	3.895	6.45736E-05
660.02	0.000066002	3.881	6.17497E-05
668.22	0.000066822	3.8675	5.90611E-05
676.42	0.000067642	3.8544	5.64965E-05
684.62	0.000068462	3.8418	5.40459E-05
692.82	0.000069282	3.8295	5.17005E-05
701.02	0.000070102	3.8177	4.94521E-05
709.22	0.000070922	3.8062	4.72934E-05
717.42	0.000071742	3.7951	4.52179E-05
725.62	7.2562E-05	3.7843	4.32198E-05
733.82	7.3382E-05	3.7738	4.12935E-05
742.02	7.4202E-05	3.7636	3.94342E-05
750.22	7.5022E-05	3.7537	3.76375E-05
758.42	7.5842E-05	3.7442	3.58993E-05
766.62	7.6662E-05	3.7348	3.42161E-05
774.82	7.7482E-05	3.7258	3.25844E-05
783.02	7.8302E-05	3.71707	3.10013E-05
791.22	7.9122E-05	3.7085	2.94641E-05
799.42	7.9942E-05	3.7002	2.79703E-05
807.62	8.0762E-05	3.69217	2.65177E-05
815.82	8.1582E-05	3.6843	2.51046E-05
824.02	8.2402E-05	3.6767	2.37292E-05
832.22	8.3222E-05	3.6693	2.23902E-05
840.42	8.4042E-05	3.66209	2.10865E-05
848.62	8.4862E-05	3.65509	1.98175E-05
856.82	8.5682E-05	3.64829	1.8583E-05
865.02	8.6502E-05	3.64168	1.73831E-05
873.22	8.7322E-05	3.63525	1.62189E-05
881.42	8.8142E-05	3.629	1.50923E-05
889.62	8.8962E-05	3.62296	1.40064E-05
897.82	8.9782E-05	3.61708	1.29665E-05
906.02	9.0602E-05	3.61138	1.19807E-05
914.22	9.1422E-05	3.60586	1.10611E-05
922.42	9.2242E-05	3.60051	1.02266E-05
930.62	9.3062E-05	3.59534	9.50441E-06
938.82	9.3882E-05	3.59053	8.93195E-06

947.02	9.4702E-05	3.58555	8.55212E-06
955.22	9.5522E-05	3.580955	8.39488E-06
963.42	9.6342E-05	3.57658	8.31825E-06
971.62	9.7162E-05	3.5724	8.24379E-06
979.82	9.7982E-05	3.56837	8.17144E-06
988.02	9.8802E-05	3.56449	8.10112E-06
996.22	9.9622E-05	3.56074	8.03277E-06
1004.42	0.000100442	3.55712	7.96632E-06
1012.62	0.000101262	3.55362	7.90168E-06

Table 10 : ISO 9845-1, 1992 : Standard Table for Reference Solar Spectral Irradiance at Air Mass 1.5: Direct Normal and Hemispherical for a 37 Degree Tilted Surface

Wavelength(nm)	*Air mass radiation*	*Wavelength(nm)*	*Air mass radiation*
300	0	570	1501.5
305	9.5	575	1475
310	42.3	580	1448.5
315	107.8	585	1422
320	181	590	1395.5
325	246	595	1417.95
330	395.3	600	1440.4
335	390.1	605	1462.85
340	435.3	610	1485.3
345	438.9	615	1472.5
350	483.7	620	1459.7
355	502	625	1446.9
360	520.3	630	1434.1
365	593.25	635	1430.55
370	666.2	640	1427
375	689.35	645	1423.45
380	712.5	650	1419.9
385	716.6	655	1413
390	720.7	660	1406.1
395	866.9	665	1399.2
400	1013.1	670	1392.3
405	1085.65	675	1326.72
410	1158.2	680	1261.15
415	1171.1	685	1195.58
420	1184	690	1130
425	1127.95	695	1176.68
430	1071.9	700	1223.35
435	1186.95	705	1270.03
440	1302	710	1316.7
445	1414	715	1125.2
450	1526	720	1020.58
455	1562.8	725	1049.66
460	1599.6	730	1103.51
465	1590.3	735	1157.35
470	1581	740	1211.2
475	1604.65	745	1204.28
480	1628.3	750	1197.36
485	1583.75	755	1184.7
490	1539.2	760	909.3
495	1543.95	765	836.9
500	1548.7	770	1050.78
505	1567.6	775	1090.94
510	1586.5	780	1131.1
515	1535.7	785	1118.72
520	1484.9	790	1106.35
525	1528.65	795	1093.97
530	1572.4	800	1081.6
535	1561.55	805	1008.97
540	1550.7	810	936.35
545	1556.1	815	863.725
550	1561.5	820	815.849
555	1546.5	825	806.9
560	1531.5	830	891.131
565	1516.5	835	934.312

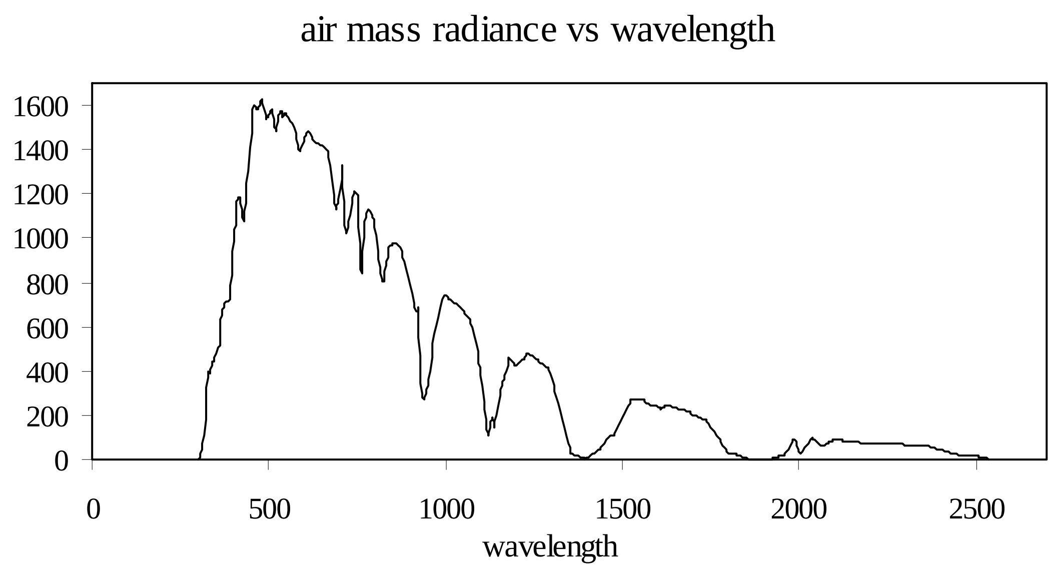
Wavelength(nm)	*Air mass radiation*	*Wavelength(nm)*	*Air mass radiation*
835	934.312	1115	184.825
840	959.9	1120	108.9
845	964.65	1125	149
850	969.4	1130	189.1
855	974.15	1135	148.457
860	978.9	1140	158.05
865	967.475	1145	201.133
870	956.05	1150	244.217
875	944.625	1155	287.3
880	933.2	1160	330.383
885	896.26	1165	364.474
890	859.32	1170	396.316
895	822.38	1175	428.158
900	785.44	1180	460
905	748.5	1185	450.9
910	708	1190	441.8
915	667.5	1195	432.7
920	678.9	1200	423.6
925	690.3	1205	431.729
930	403.6	1210	439.857
935	299.814	1215	447.986
940	273.382	1220	456.114
945	298.518	1225	464.243
950	338.682	1230	472.371
955	401.388	1235	480.5
960	464.094	1240	474.373
965	526.8	1245	468.245
970	566.667	1250	462.118
975	606.533	1255	455.991
980	646.4	1260	449.864
985	683.585	1265	443.736
990	720.77	1270	437.609
995	744.984	1275	431.482
1000	738.93	1280	425.355
1005	732.876	1285	419.227
1010	726.823	1290	413.1
1015	720.769	1295	385.95
1020	714.715	1300	358.8
1025	708.661	1305	331.65
1030	702.608	1310	304.5
1035	696.554	1315	277.35
1040	690.5	1320	250.2
1045	681.667	1325	213.917
1050	672.833	1330	177.633
1055	664	1335	141.35
1060	655.167	1340	105.067
1065	646.333	1345	68.7833
1070	637.5	1350	32.5
1075	600.017	1355	29.0667
1080	562.533	1360	25.6333
1085	525.05	1365	22.2
1090	487.567	1370	18.7667
1095	450.083	1375	15.3333
1100	412.6	1380	11.9
1105	336.675	1385	8.46667
1110	260.75	1390	5.03333

Wavelength(nm)	*Air mass radiation*	*Wavelength(nm)*	*Air mass radiation*
1395	1.6	1675	221.841
1400	7.29474	1680	218.919
1405	12.9895	1685	214.968
1410	18.6842	1690	211.016
1415	24.3789	1695	207.065
1420	30.0737	1700	203.113
1425	35.7684	1705	199.161
1430	41.4632	1710	195.21
1435	47.1579	1715	191.258
1440	52.8526	1720	187.306
1445	61.875	1725	183.355
1450	74.225	1730	179.403
1455	86.575	1735	175.452
1460	98.925	1740	171.5
1465	105.169	1745	159.767
1470	105.307	1750	148.033
1475	105.445	1755	136.3
1480	116.99	1760	124.567
1485	136.14	1765	112.833
1490	155.29	1770	101.1
1495	174.44	1775	89.3667
1500	192.548	1780	77.6333
1505	209.961	1785	65.9
1510	227.374	1790	54.1667
1515	244.787	1795	42.4333
1520	262.2	1800	30.7
1525	265.358	1805	28.3083
1530	268.516	1810	25.9167
1535	271.674	1815	23.525
1540	274.242	1820	21.1333
1545	274.453	1825	18.7417
1550	274.663	1830	16.35
1555	274.874	1835	13.9583
1560	271.96	1840	11.5667
1565	264.36	1845	9.175
1570	256.76	1850	6.78333
1575	249.16	1855	4.39167
1580	245	1860	2
1585	246	1865	1.93333
1590	247	1870	1.86667
1595	244.283	1875	1.8
1600	239.089	1880	1.73333
1605	233.894	1885	1.66667
1610	228.7	1890	1.6
1615	232.65	1895	1.53333
1620	236.6	1900	1.46667
1625	240.55	1905	1.4
1630	244.5	1910	1.33333
1635	241.469	1915	1.26667
1640	238.438	1920	1.2
1645	235.406	1925	3.7
1650	233.013	1930	6.2
1655	230.778	1935	8.7
1660	228.544	1940	11.2
1665	226.309	1945	13.7
1670	224.075	1950	16.2

Wavelength(nm)	*Air mass radiation*	*Wavelength(nm)*	*Air mass radiation*
1955	18.7	2235	70.8319
1960	21.2	2240	70.7417
1965	35.18	2245	70.6514
1970	49.16	2250	70.5611
1975	63.14	2255	70.4708
1980	77.12	2260	70.3806
1985	91.1	2265	70.2903
1990	75.025	2270	70.2
1995	58.95	2275	69.7444
2000	42.875	2280	69.2889
2005	26.8	2285	68.8333
2010	38.9167	2290	68.3778
2015	51.0333	2295	67.9222
2020	63.15	2300	67.4667
2025	75.2667	2305	67.0111
2030	87.3833	2310	66.5556
2035	99.5	2315	66.1
2040	92.9833	2320	65.6444
2045	86.4667	2325	65.1889
2050	79.95	2330	64.7333
2055	73.4333	2335	64.2778
2060	66.9167	2340	63.8222
2065	60.4	2345	63.3667
2070	64.5	2350	62.9111
2075	68.6	2355	62.4556
2080	72.7	2360	62
2085	76.8	2365	59.7333
2090	80.9	2370	57.4667
2095	85	2375	55.2
2100	89.1	2380	52.9333
2105	88.3812	2385	50.6667
2110	87.6625	2390	48.4
2115	86.9437	2395	46.1333
2120	86.225	2400	43.8667
2125	85.5062	2405	41.6
2130	84.7875	2410	39.3333
2135	84.0687	2415	37.0667
2140	83.35	2420	34.8
2145	82.6313	2425	32.5333
2150	81.772	2430	30.2667
2155	80.702	2435	28
2160	79.632	2440	25.7333
2165	78.562	2445	23.4667
2170	77.492	2450	21.2
2175	76.422	2455	20.8932
2180	75.352	2460	20.5864
2185	74.282	2465	20.2795
2190	73.212	2470	19.9727
2195	72.142	2475	19.6659
2200	71.4639	2480	19.3591
2205	71.3736	2485	19.0523
2210	71.2833	2490	18.7455
2215	71.1931	2495	18.1268
2220	71.1028	2500	16.261
2225	71.0125	2505	14.3951
2230	70.9222	2510	12.5293

Wavelength(nm)	*Air mass radiation*	*Wavelength(nm)*	*Air mass radiation*
2515 2520 2525 2530 2535	10.6634 8.79756 6.93171 5.06585 3.2		

The graph of above data set is,



Graph 23 **:** *Reference Solar Spectral Irradiance at Air Mass 1.5: Direct Normal and Hemispherical for a 37 Degree Tilted Surface*

Gauss elimination method

One of the most popular techniques for solving simultaneous linear equations is the Gaussian elimination method. The approach is designed to solve a general set of *n* equations and *n* unknowns

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + ... + a_{1n}x_n = b_1 \tag{2.143}$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + ... + a_{2n}x_n = b_2 \tag{2.144}$$

$$\begin{matrix} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{matrix}$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + ... + a_{nn}x_n = b_n \tag{2.145}$$

Gaussian elimination consists of two steps,

Forward Elimination of Unknowns: In this step, the unknown is eliminated in each equation starting with the first equation. This way, the equations are "reduced" to one equation and one unknown in each equation.

Back Substitution: In this step, starting from the last equation, each of the unknowns is found.

a) Forward Elimination of Unknowns:

In the first step of forward elimination, the first unknown, x_1 is eliminated from all rows below the first row. The first equation is selected as the pivot equation to eliminate x_1. So, to eliminate x_1 in the second equation, one divides the first equation by a_{11} (hence called the pivot element) and then multiply it by a_{21}. That is, same as multiplying the first equation by a_{21}/ a_{11} to give ,

$$a_{21}x_1 + \frac{a_{21}}{a_{11}}a_{12}x_2 + ... + \frac{a_{21}}{a_{11}}a_{1n}x_n = \frac{a_{21}}{a_{11}}b_1 \tag{2.146}$$

Now, this equation can be subtracted from the second equation to give

$$\left(a_{22} - \frac{a_{21}}{a_{11}}a_{12}\right)x_2 + ... + \left(a_{2n} - \frac{a_{21}}{a_{11}}a_{1n}\right)x_n = b_2 - \frac{a_{21}}{a_{11}}b_1 \tag{2.147}$$

or

$$a'_{22}x_2 + ... + a'_{2n}x_n = b'_2 \tag{2.148}$$

where

$$\begin{aligned} a'_{22} &= a_{22} - \frac{a_{21}}{a_{11}}a_{12} \\ &\vdots \\ a'_{2n} &= a_{2n} - \frac{a_{21}}{a_{11}}a_{1n} \end{aligned} \tag{2.149}$$

This procedure of eliminating x_1, is now repeated for the third equation to the n[th] equation to reduce the set of equations as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \tag{2.150}$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2 \tag{2.151}$$

$$a'_{32}x_2 + a'_{33}x_3 + \ldots + a'_{3n}x_n = b'_3 \tag{2.152}$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$a'_{n2}x_2 + a'_{n3}x_3 + \ldots + a'_{nn}x_n = b'_n \tag{2.153}$$

This is the end of the first step of forward elimination. Now for the second step of forward elimination, it is started with the second equation as the pivot equation and a'_{22} as the pivot element. So, to eliminate x_2 in the third equation, one divides the second equation by a'_{22} (the pivot element) and then multiply it by a'_{32}. That is, same as multiplying the second equation by a'_{32}/a'_{22} and subtracting from the third equation. This makes the coefficient of x_2 zero in the third equation. The same procedure is now repeated for the fourth equation till the n[th] equation to give

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \tag{2.154}$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2 \tag{2.155}$$

$$a''_{33}x_3 + \ldots + a''_{3n}x_n = b''_3 \tag{2.156}$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$a''_{n3}x_3 + \ldots + a''_{nn}x_n = b''_n \tag{2.157}$$

The next steps of forward elimination are conducted by using the third equation as a pivot equation and so on. That is, there will be a total of (n-1) steps of forward elimination. At the end of (n-1) steps of forward elimination, we get a set of equations that look like

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1 \tag{2.158}$$

$$a'_{22}x_2 + a'_{23}x_3 + \ldots + a'_{2n}x_n = b'_2 \tag{2.159}$$

$$a''_{33}x_3 + \ldots + a''_{3n}x_n = b''_3 \tag{2.160}$$

$$\begin{matrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{matrix}$$

$$a_{nn}^{(n-1)}x_n = b_n^{(n-1)} \tag{2.161}$$

b) Back Substitution:

Now the equations are solved starting from the last equation as it has only one unknown.

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}} \qquad (2.162)$$

Then the second last equation, that is the (n-1)th equation, has two unknowns -x_n and x_{n-1}, but x_n is already known. This reduces the (n-1)th equation also to one unknown. Back substitution hence can be represented for all equations by the formula

$$x_i = \frac{b_i^{(i-1)} - \sum_{j=i+1}^{n} a_{ij}^{(i-1)} x_j}{a_{ii}^{(i-1)}} \qquad \text{for } i = n-1,\ n-2, \ldots, \qquad (2.163)$$

And

$$x_n = \frac{b_n^{(n-1)}}{a_{nn}^{(n-1)}} \qquad (2.164)$$

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